Digital Signatures

Chethan Kamath

IITB TRUST LAB

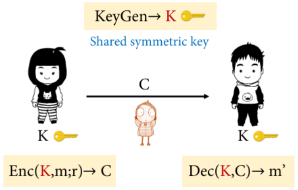
DIGITAL : SECURE : RESPONSIBLE

ACM Summer School 2024, 6/Jun/2024

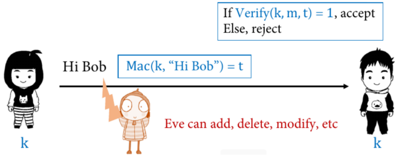
Recall from Prior Sessions

SYMMETRIC KEY UNIVERSE	PUBLIC KEY UNIVERSE

SKE



MAC



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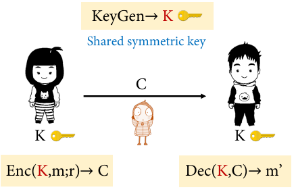

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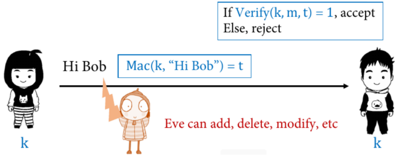
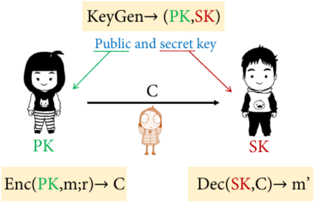


PUBLIC KEY UNIVERSE

PKE



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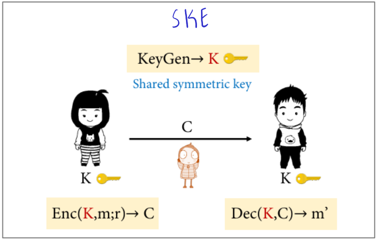

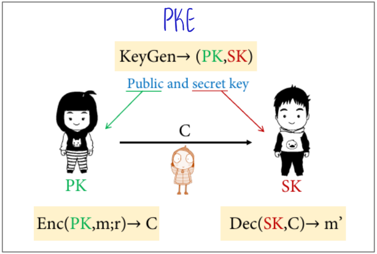

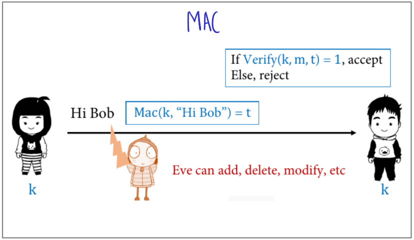

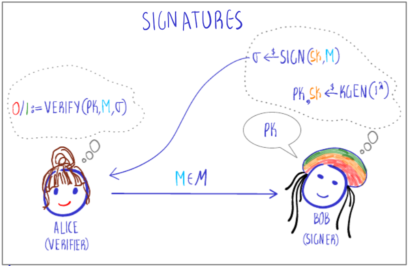

Plan for this Session

Digital Signature: Syntax and Modelling Security

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One-Time Signatures

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Many-Time (Stateful) Signatures

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Digital (Analogues of Physical) Signatures...

- Requirement: no one should be able to forge Bob's signature

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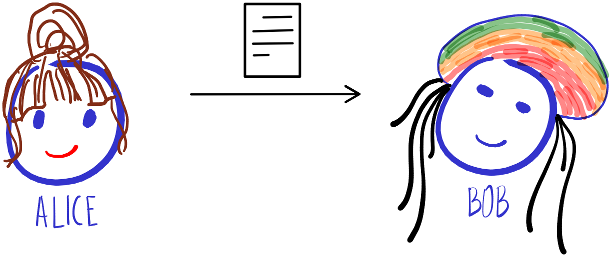

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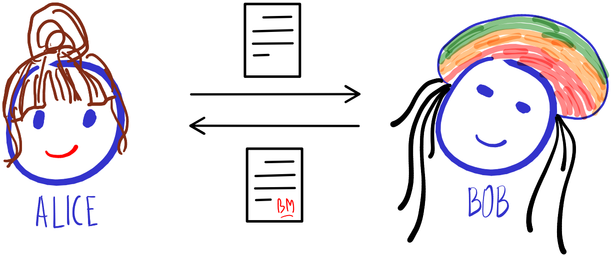

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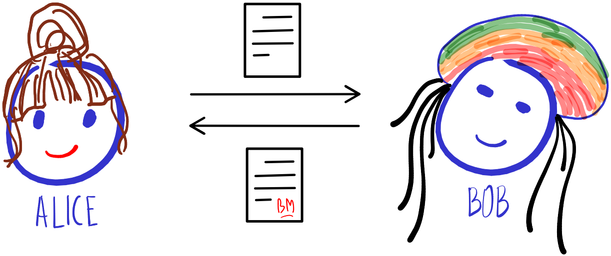

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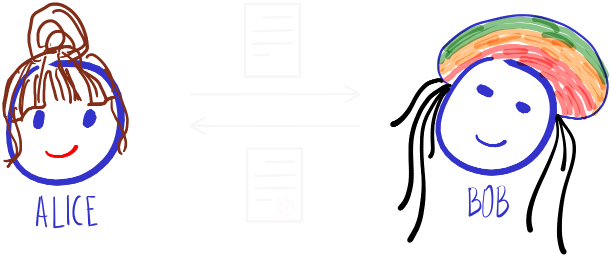

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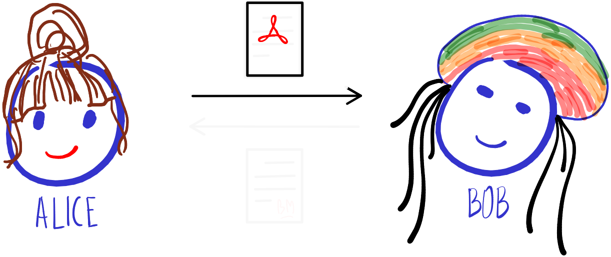

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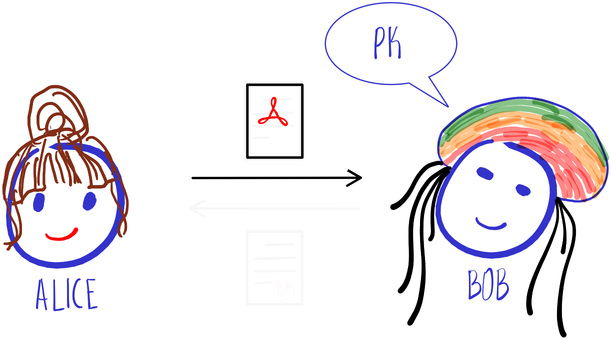

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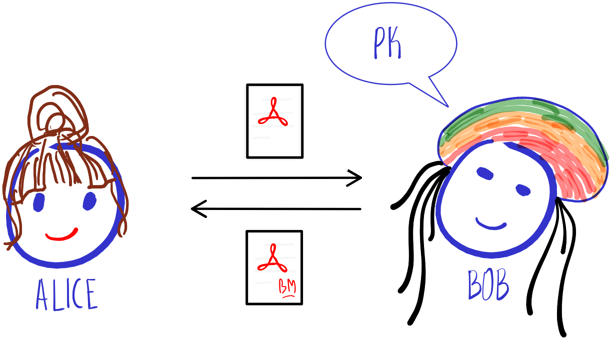

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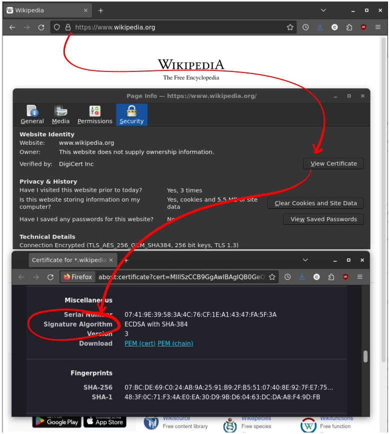

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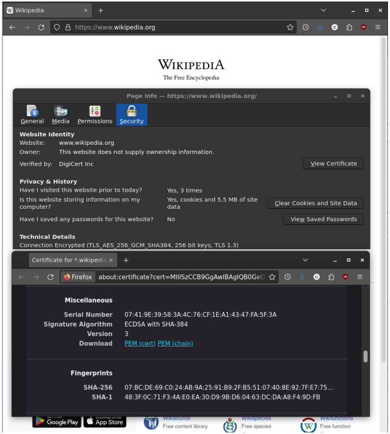

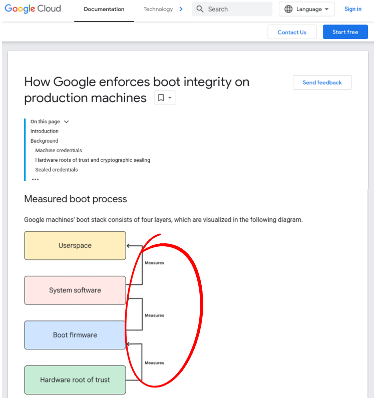

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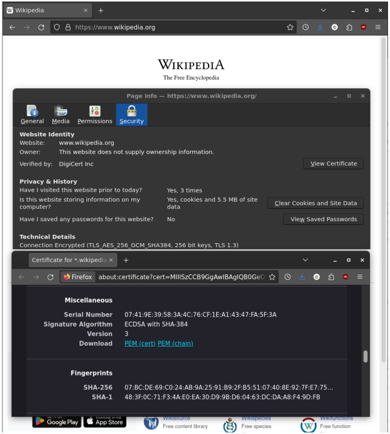

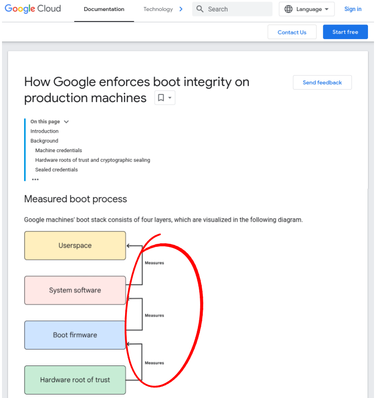

- Application to blockchains protocols like Algorand and Chia.

Syntax

- *Public-key* analogue of message authentication codes (MAC)
  - Triple of algorithms: $(\mathtt{KGEN}, \mathtt{SIGN}, \mathtt{VERIFY})$

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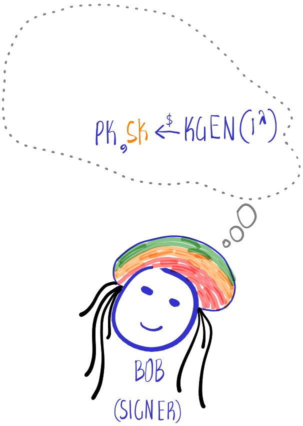

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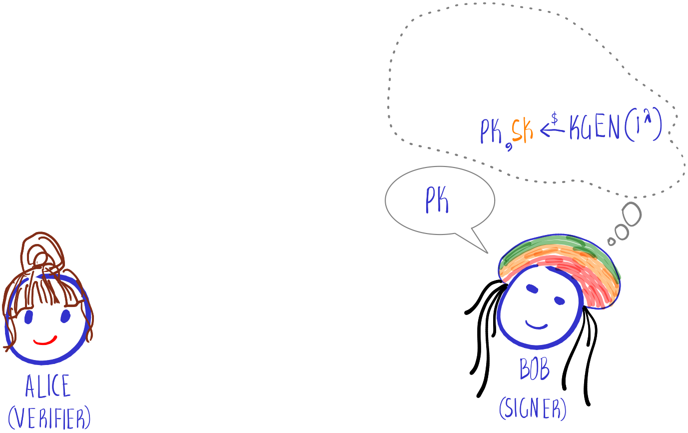

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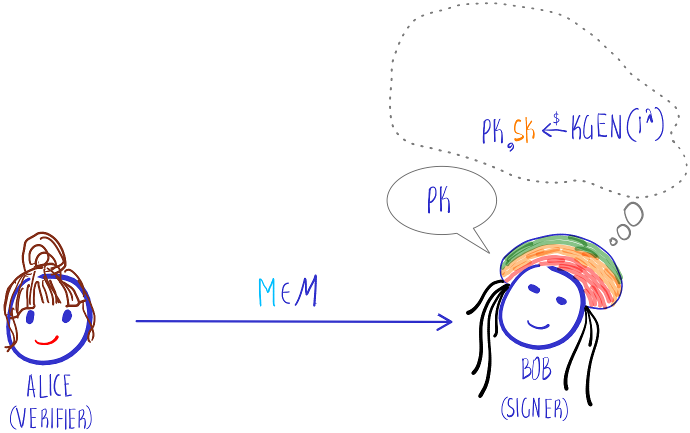

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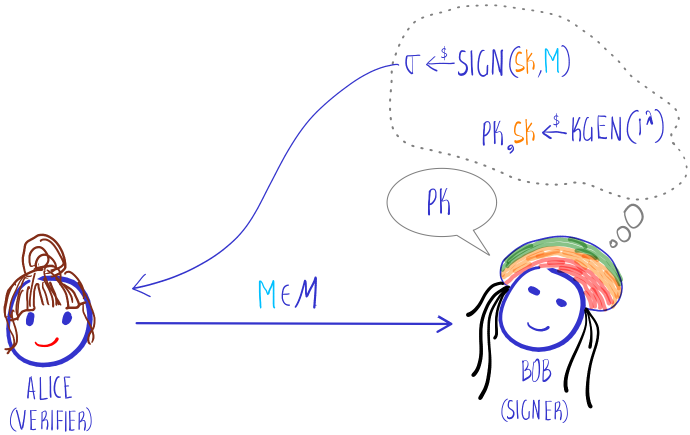

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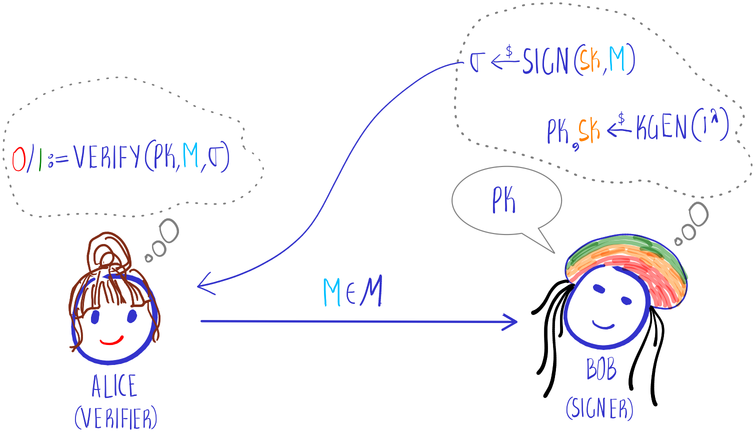

Security: Universal Unforgeability under Key-Only Attack

"BREAK" "ATTACK"

Definition 1

A scheme is UU-KOA secure if no PPT forger can break as above with a *non-negligible* probability.

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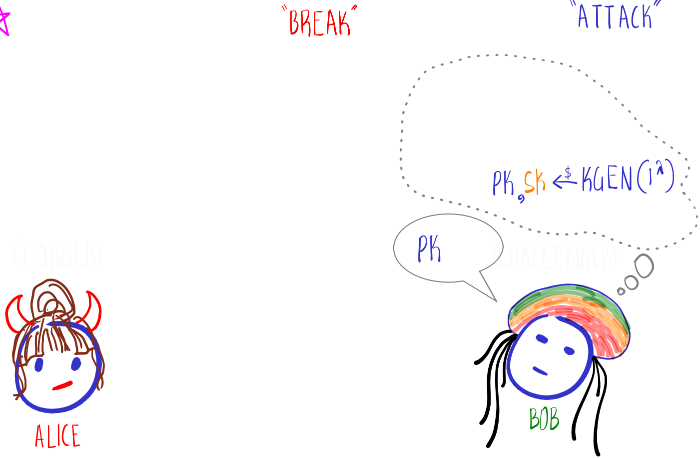

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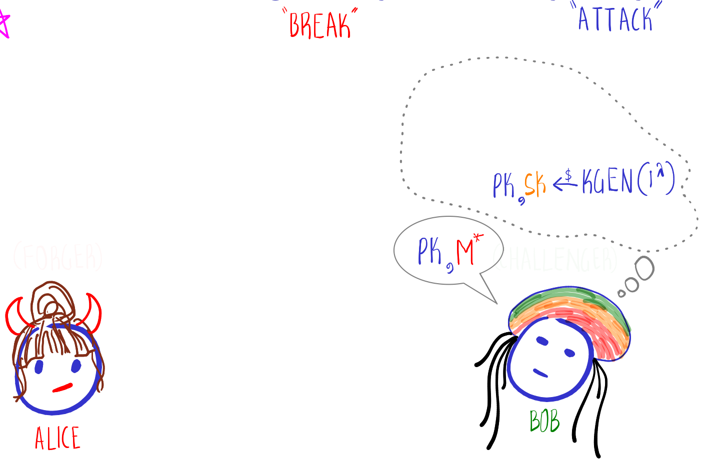

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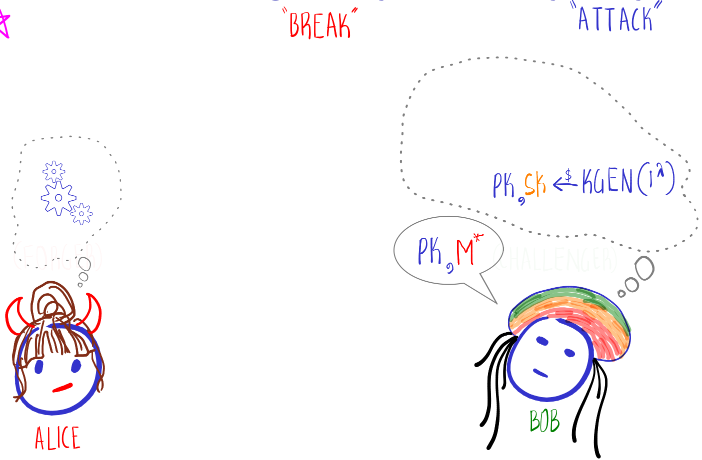

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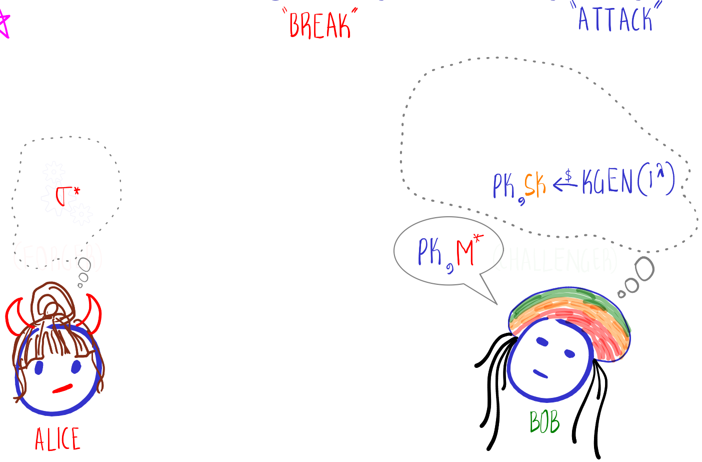

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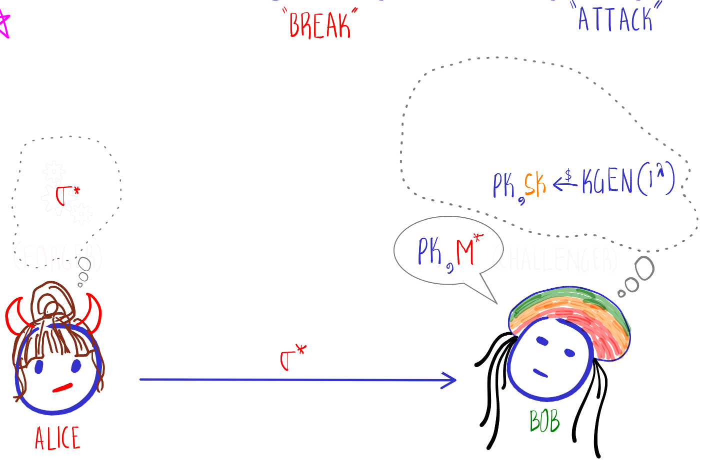

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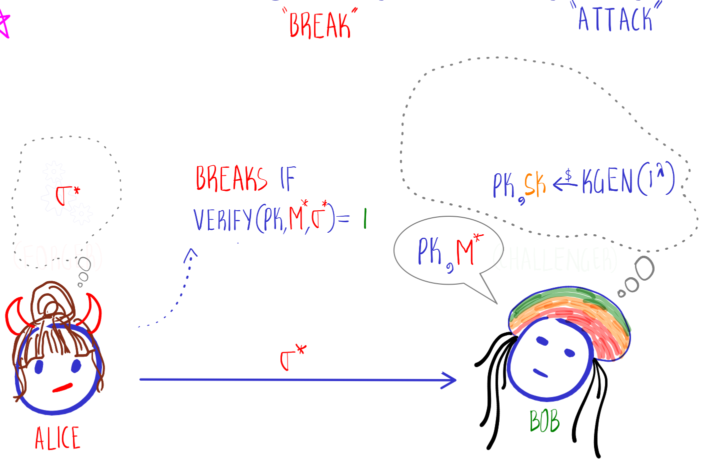

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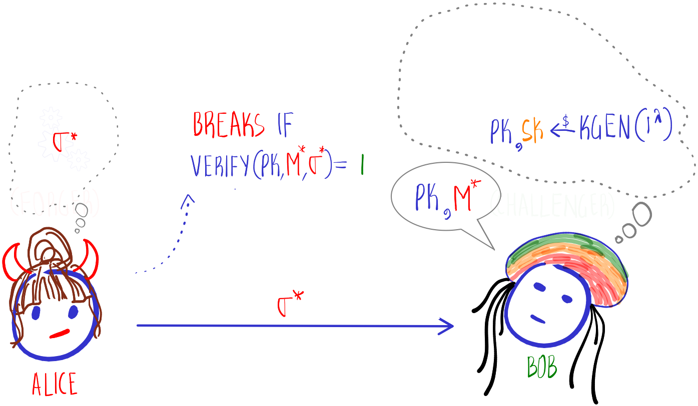

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Security: Existential Unforg. under Key-Only Attack

$\forall \rightarrow \exists$

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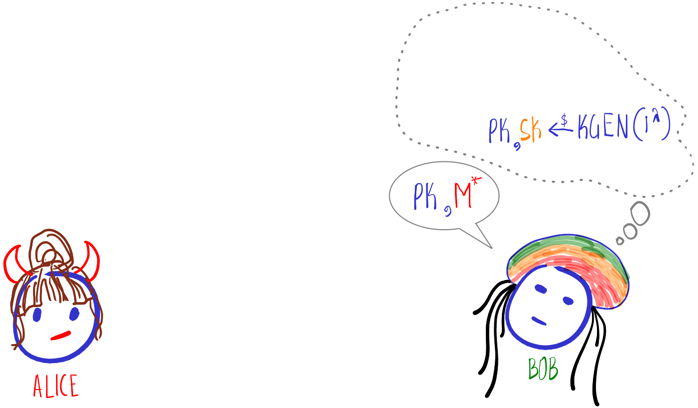

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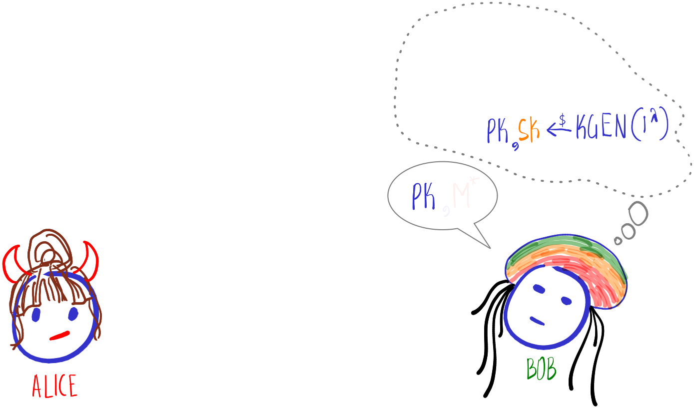

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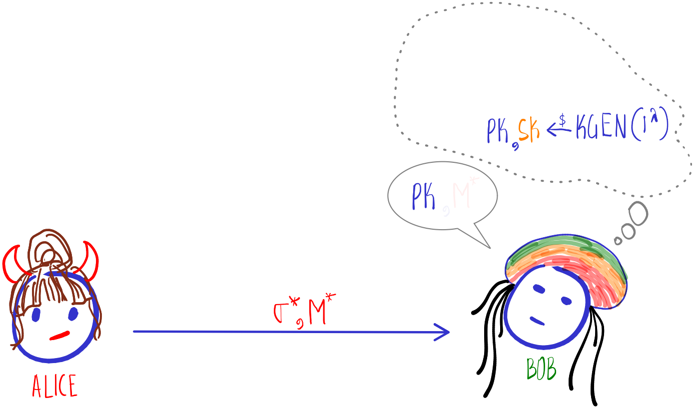

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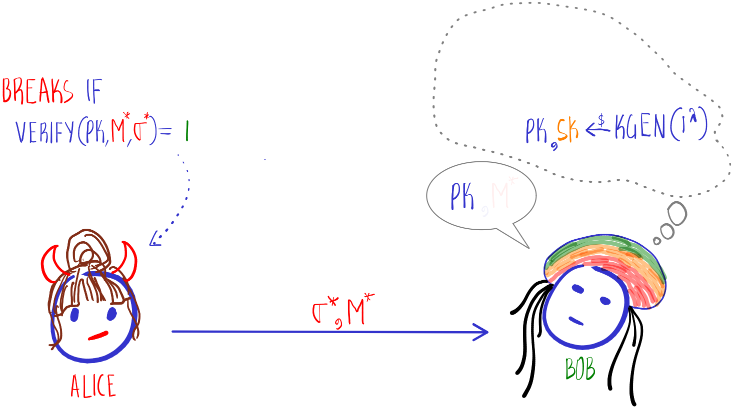

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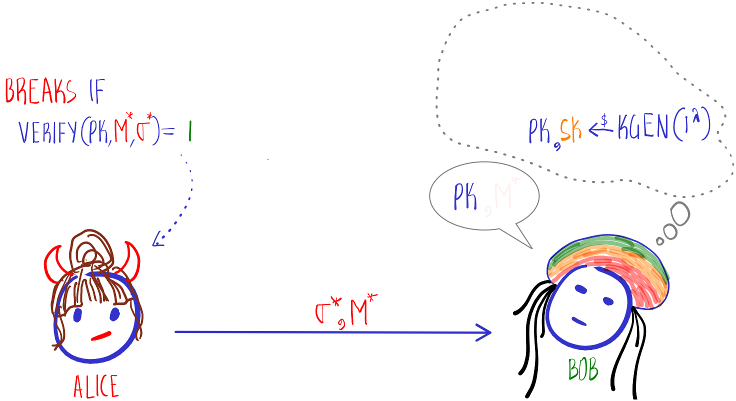

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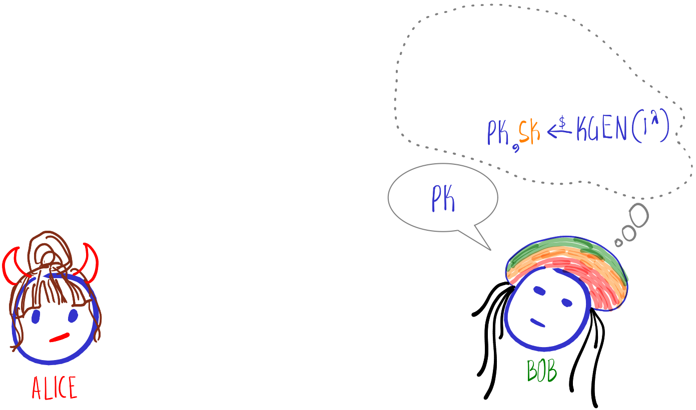

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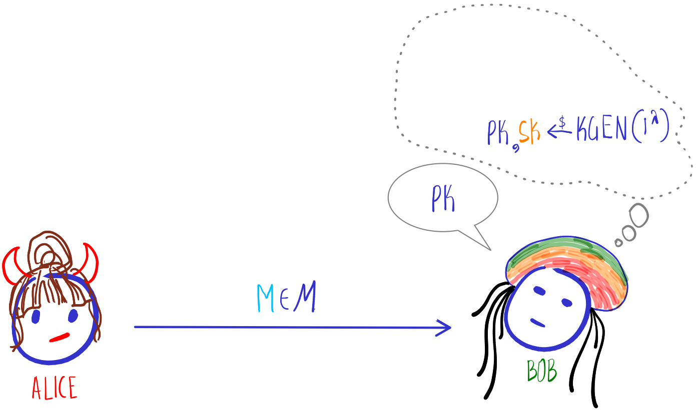

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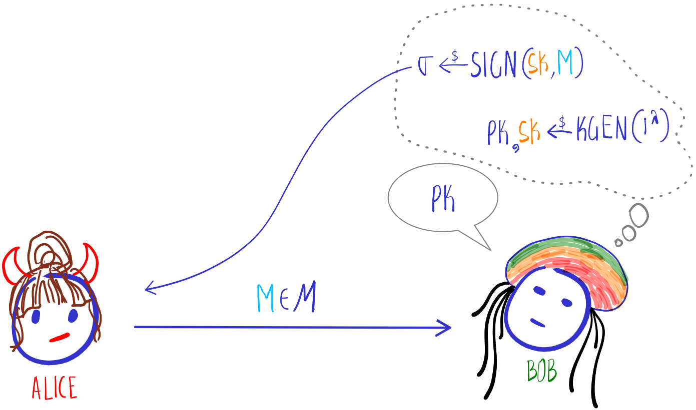

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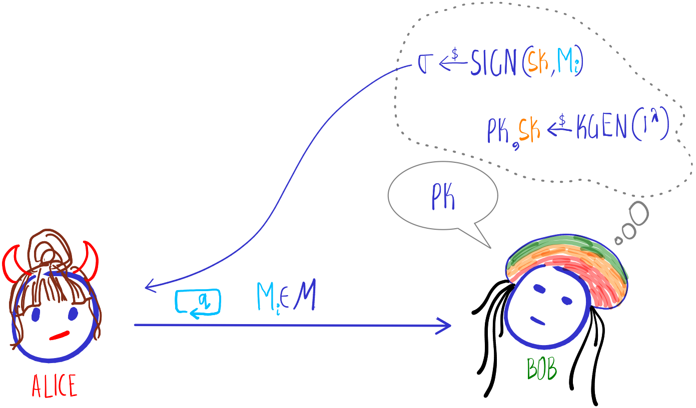

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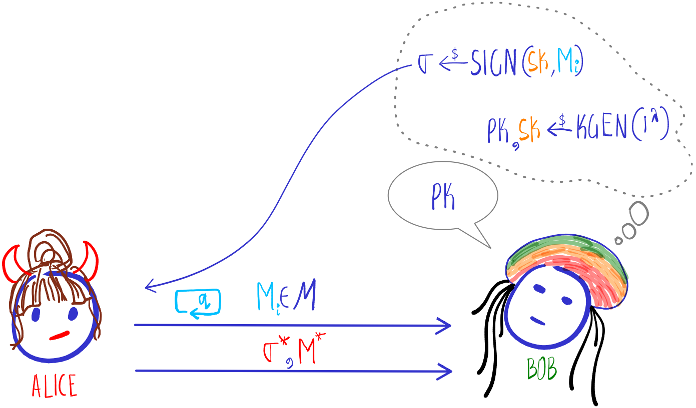

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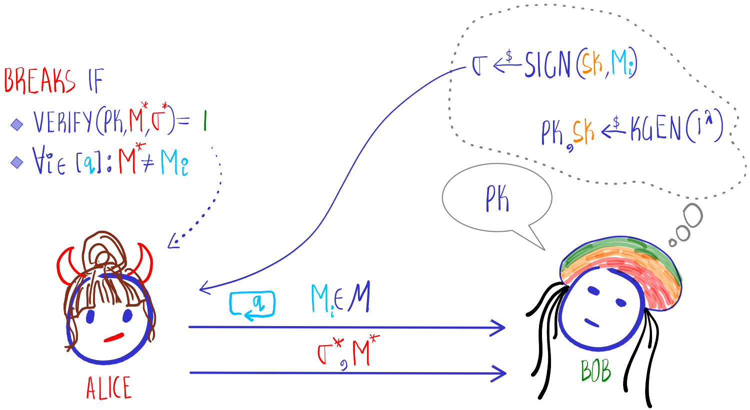

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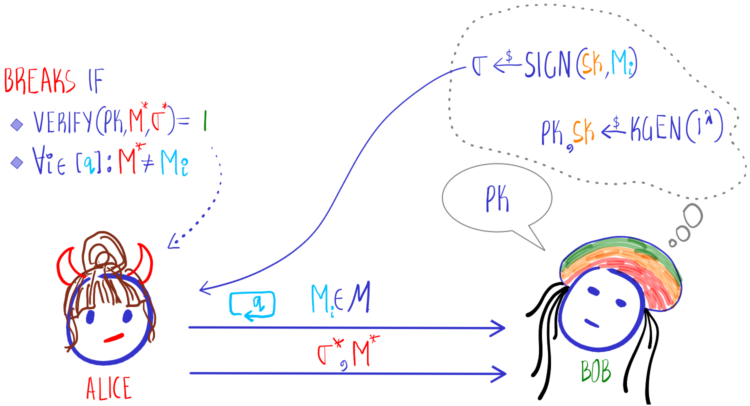

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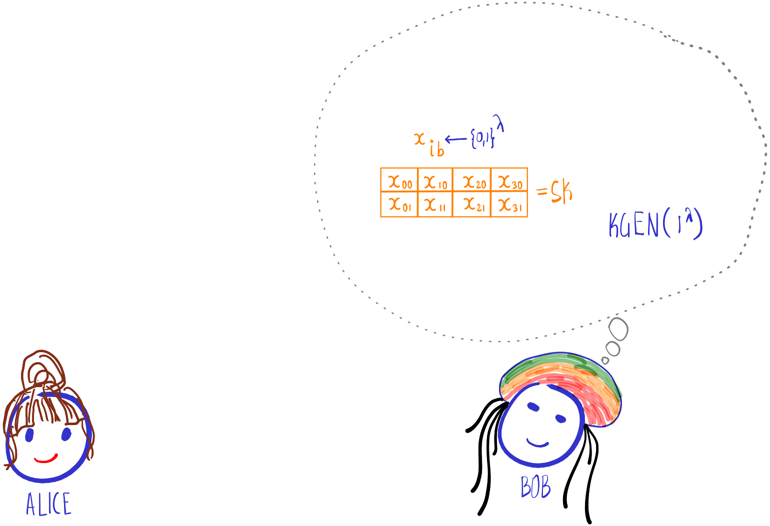

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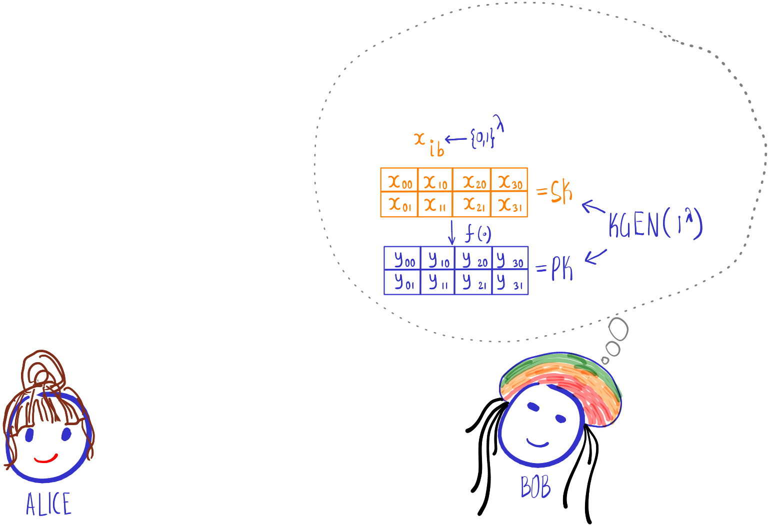

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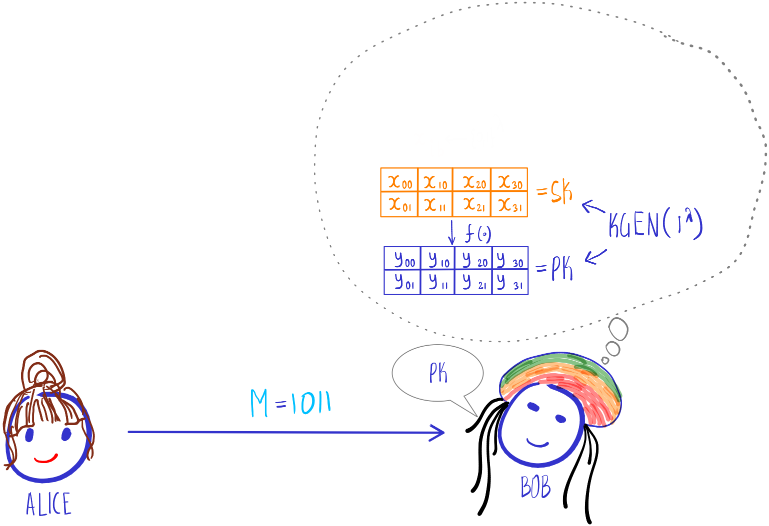

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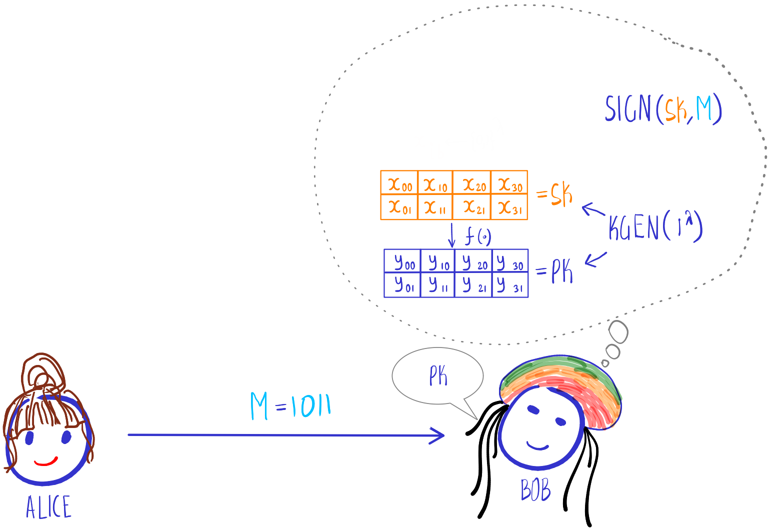

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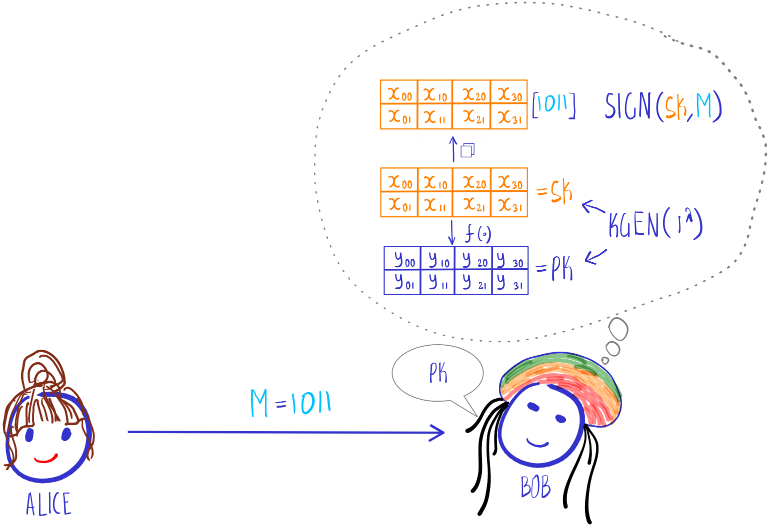

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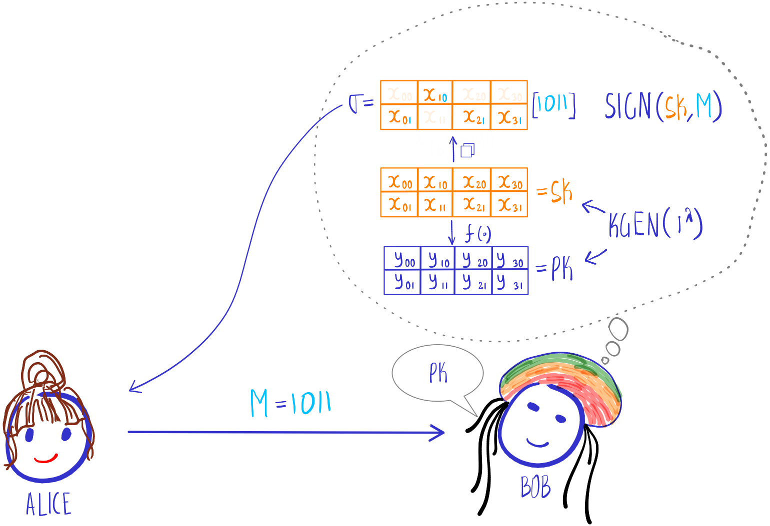

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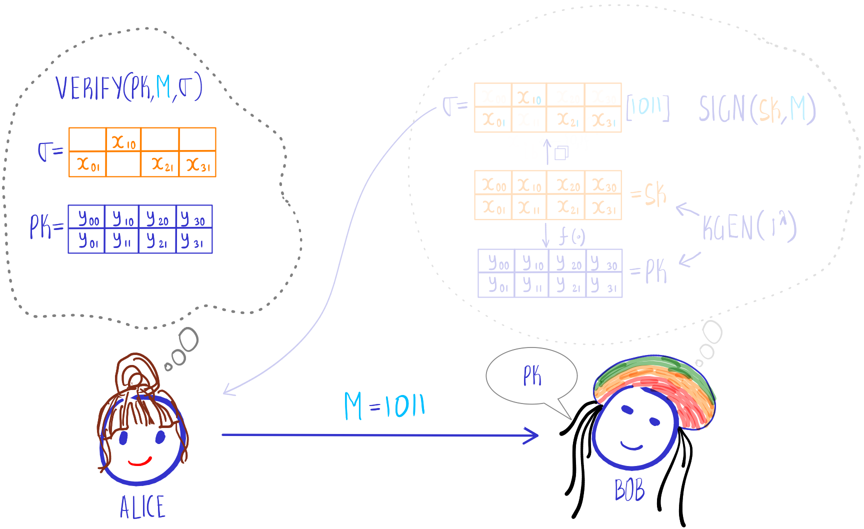

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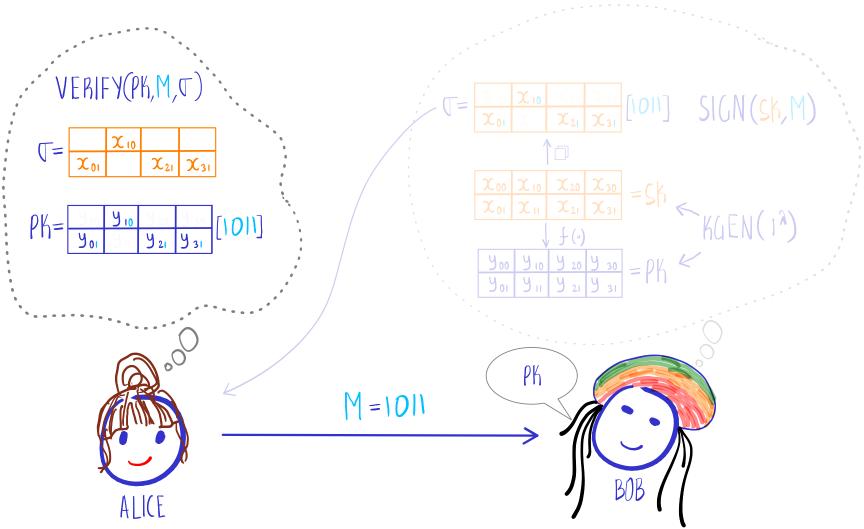

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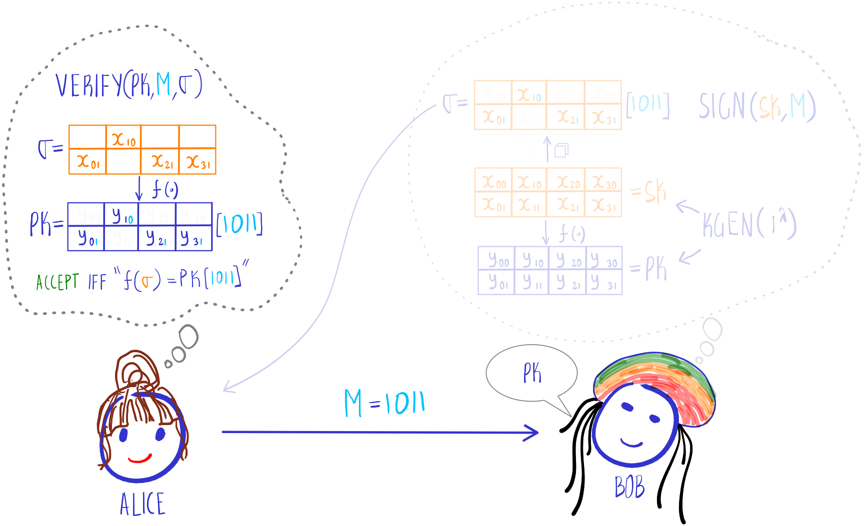

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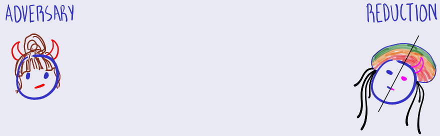

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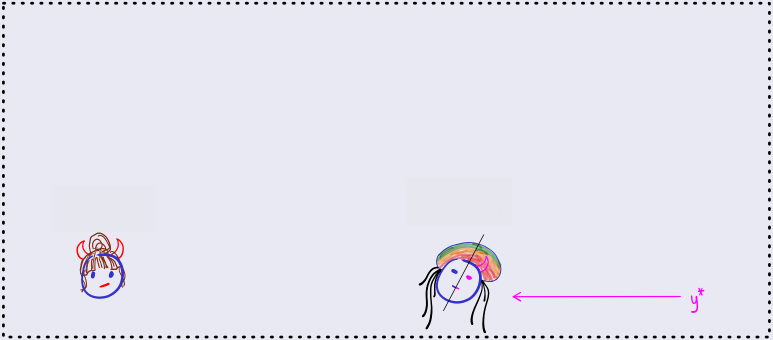

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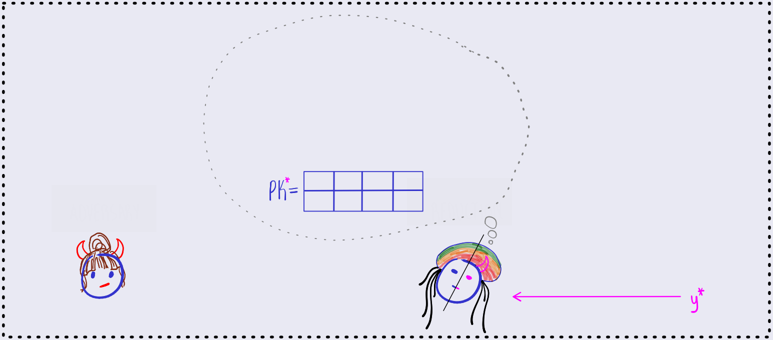

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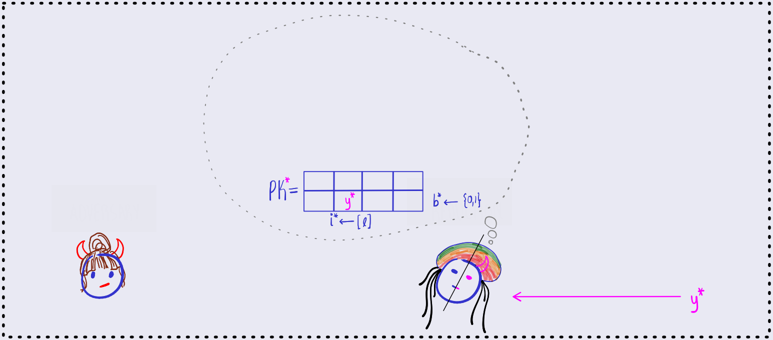

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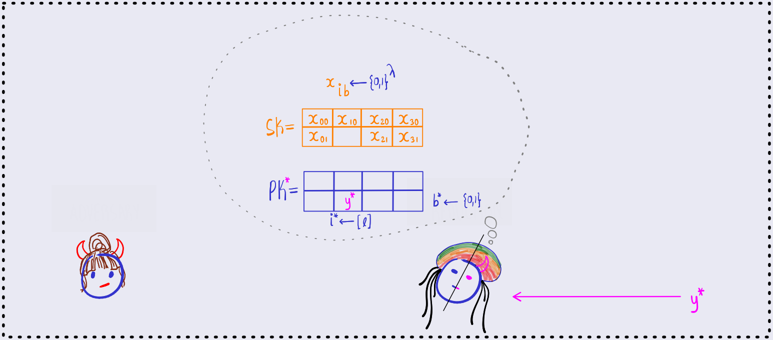

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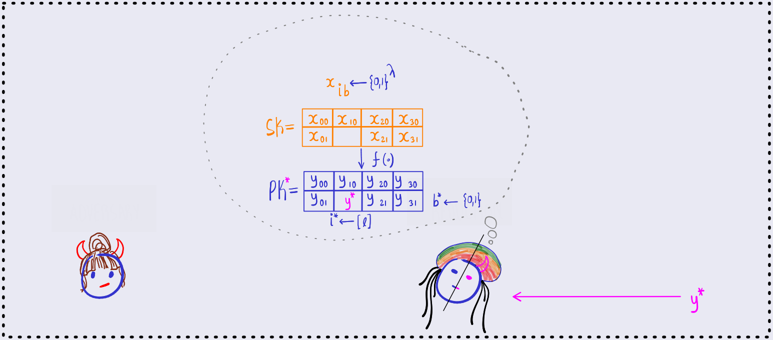

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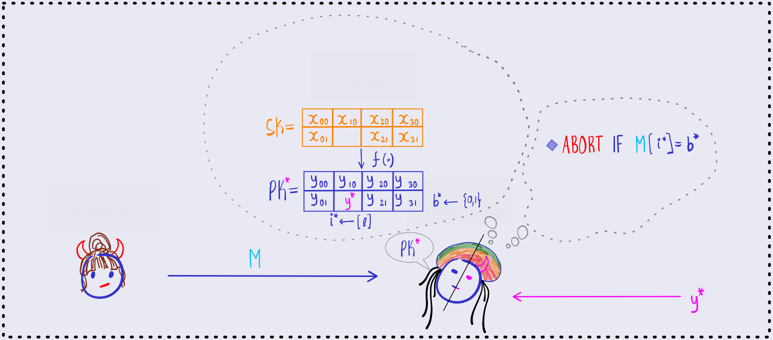

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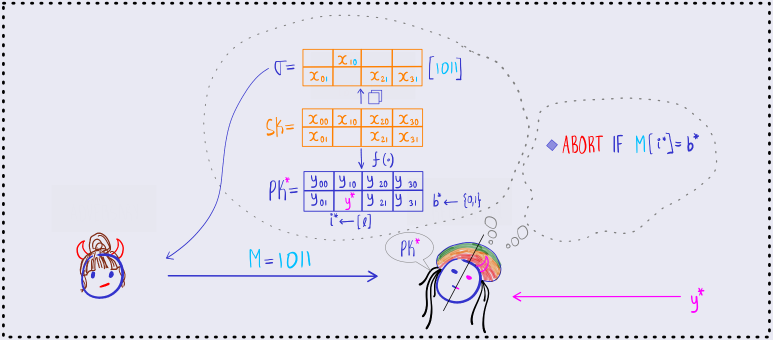

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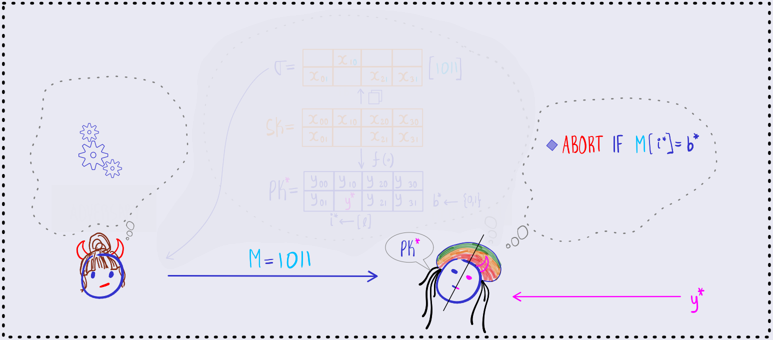

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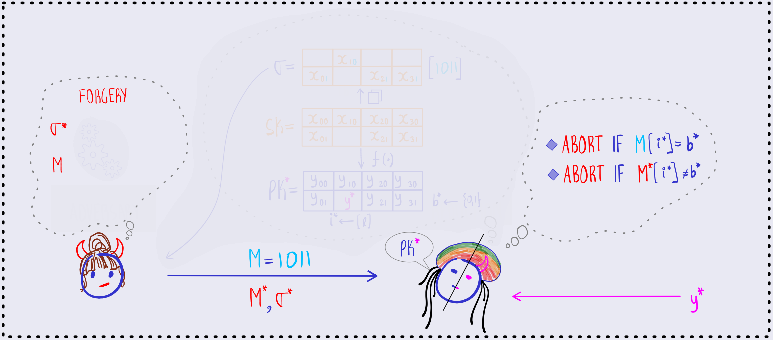

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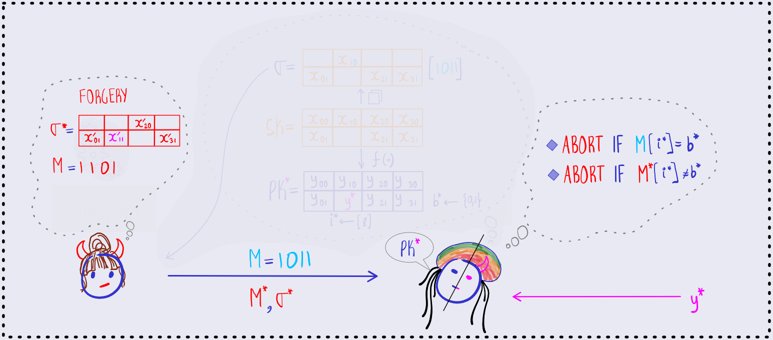

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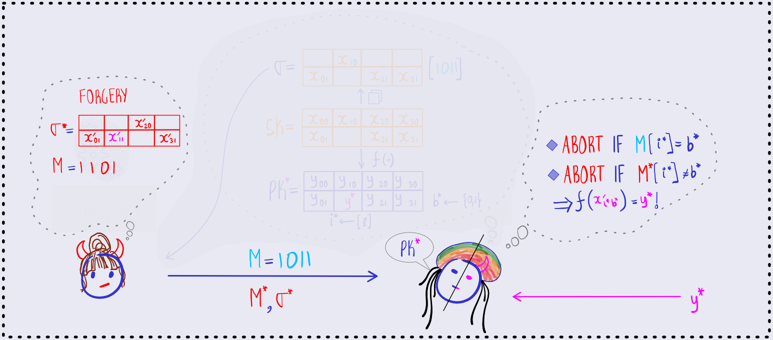

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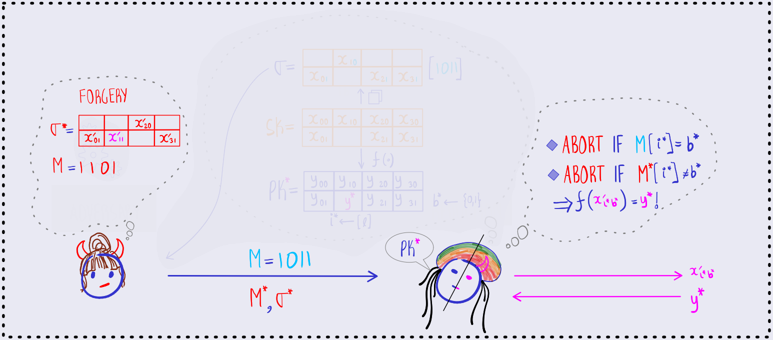

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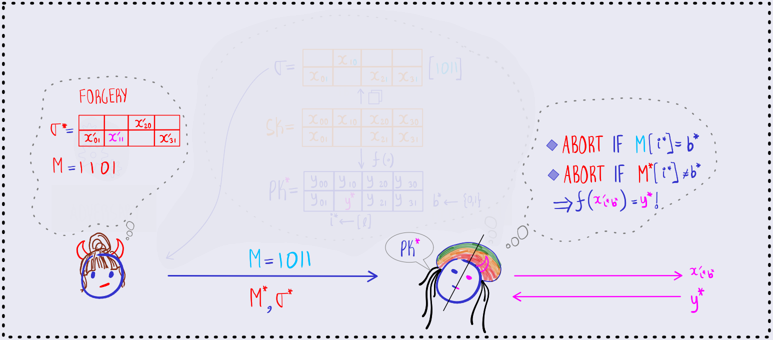

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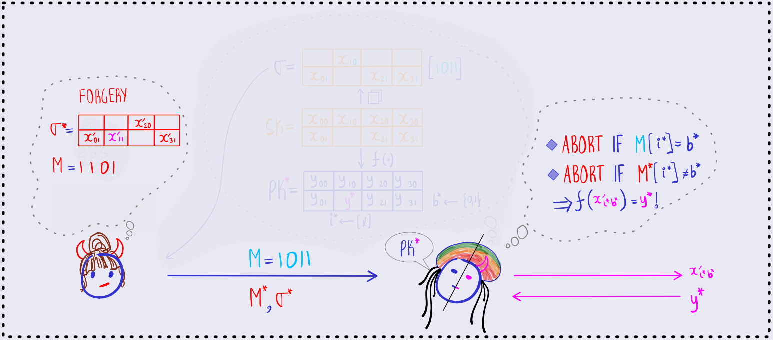

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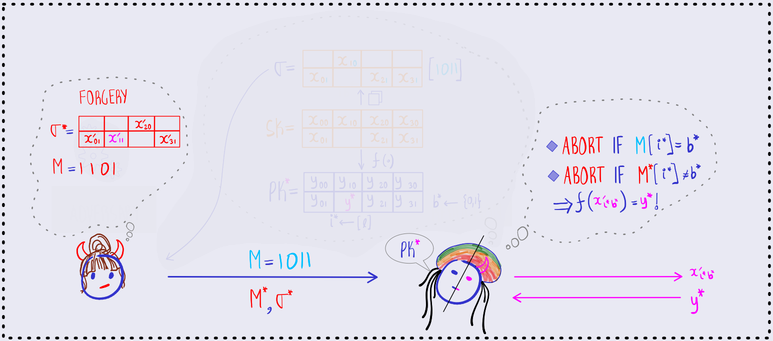

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Exercise 1

- *Can a forger break EU-CMA given two signatures?*
- *What happens if we fix $i^* = 0$ in the proof?*
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Theorem 5

If f is a OWF then Lamport's scheme is a one-time signature for fixed-length messages.

Exercise 2 (Domain Extension)

Given a collision-resistant hash function $H : \{0,1\}^{2\ell} \to \{0,1\}^{\ell}$, construct a OTS for arbitrary-length messages.

Plan for this Session

Digital Signature: Syntax and Modelling Security

One-Time Signatures

Many-Time (Stateful) Signatures

Efficient Signatures via Hash-and-Sign

Wrapping Up

(Many-Time) Signatures with Stateful Signer

- Syntax: same as before except that `SIGN` is *stateful*

- Security: same as before (i.e., forger *not* given any state)

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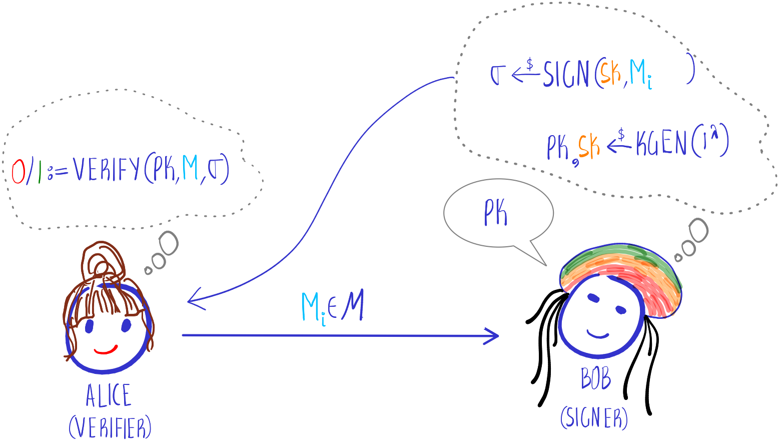

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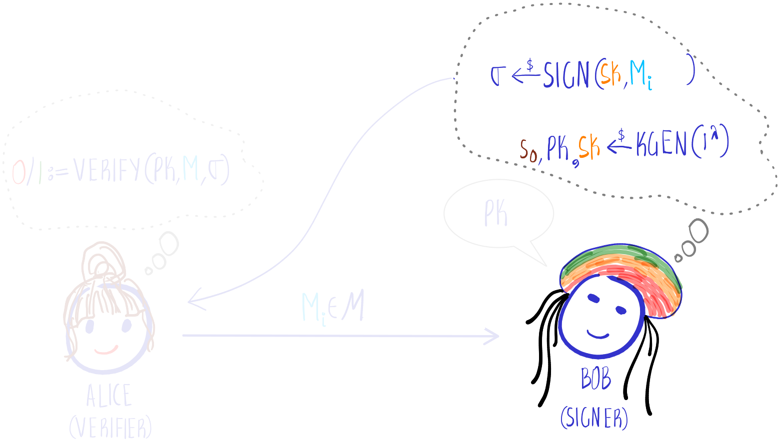

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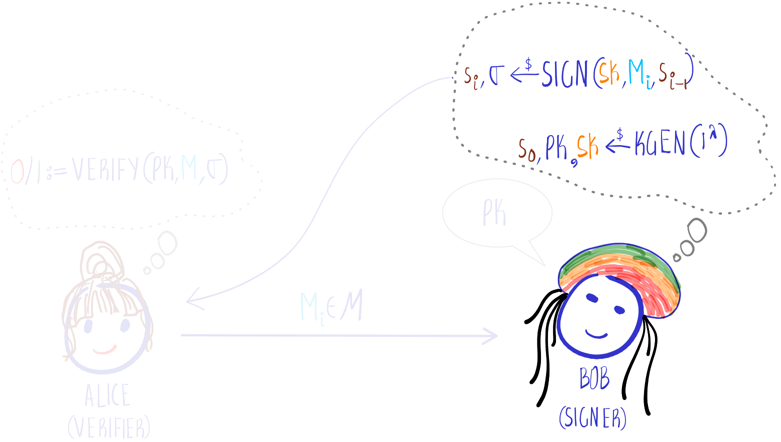

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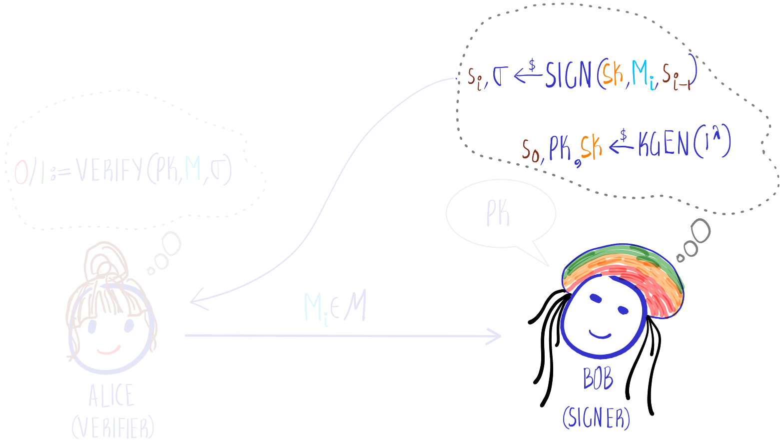

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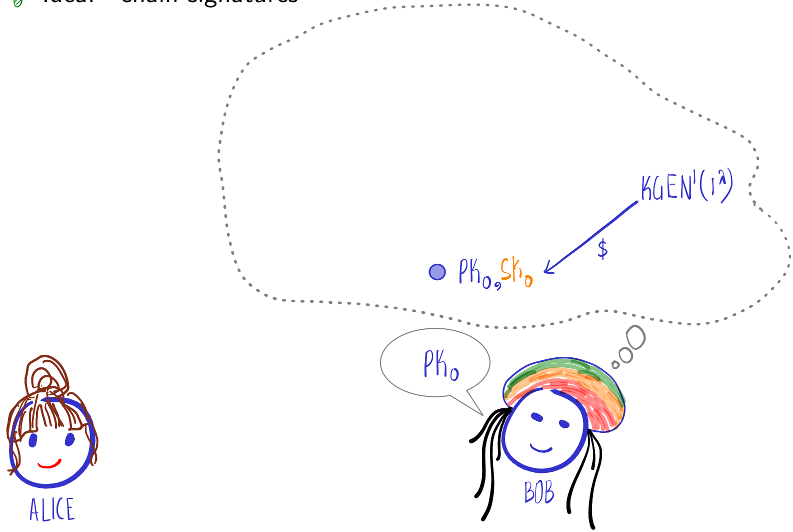

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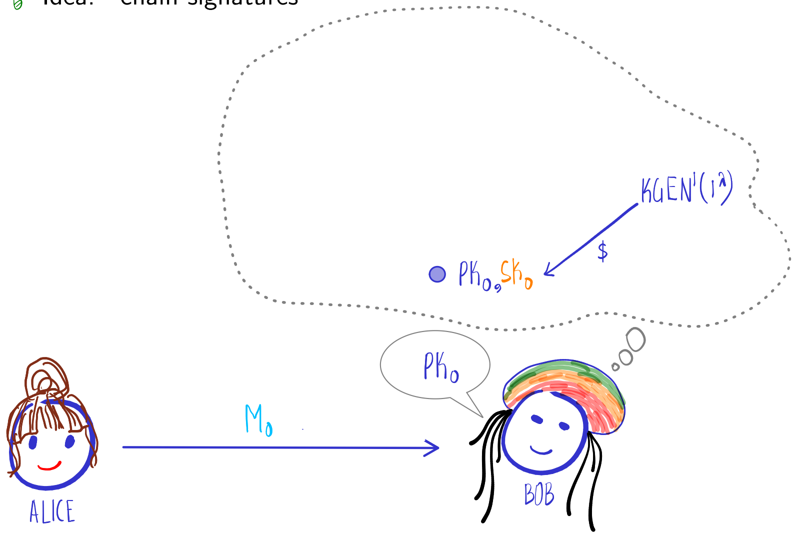

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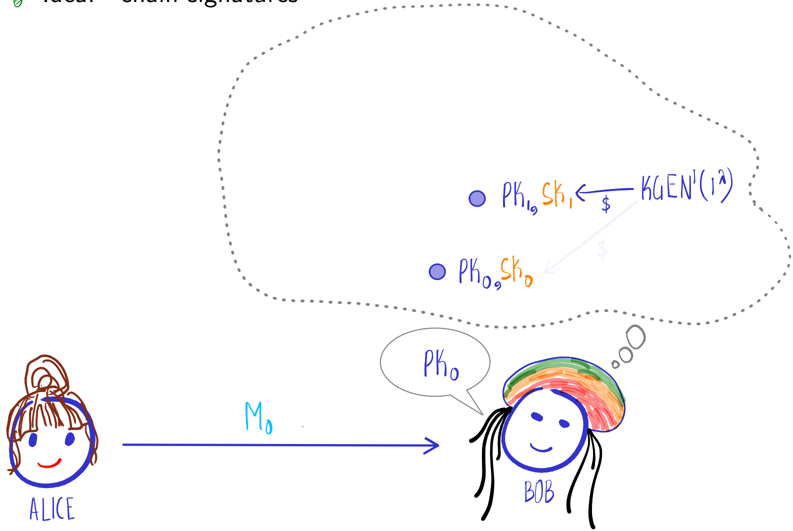

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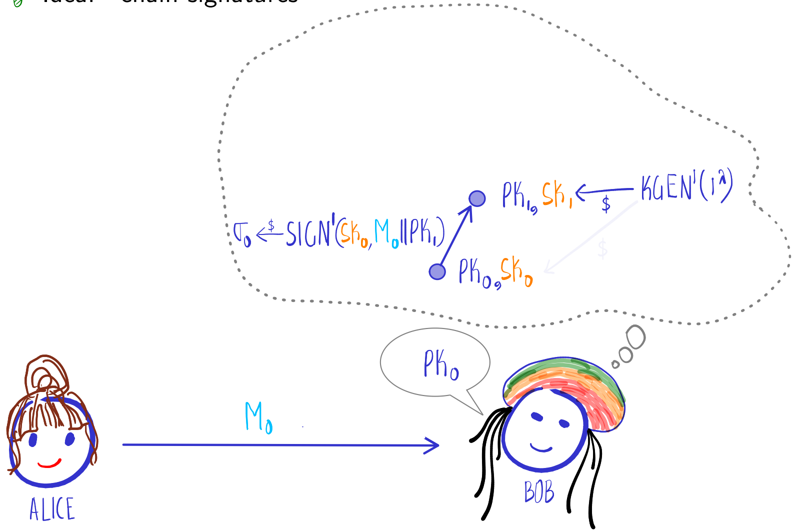

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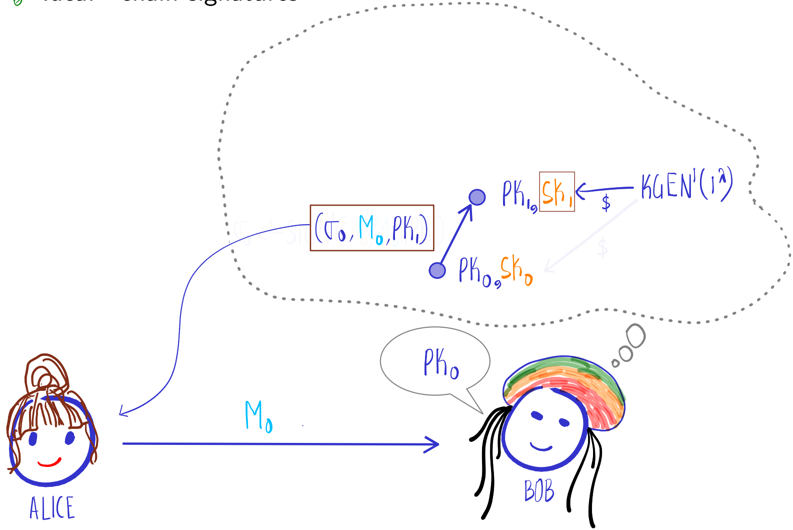

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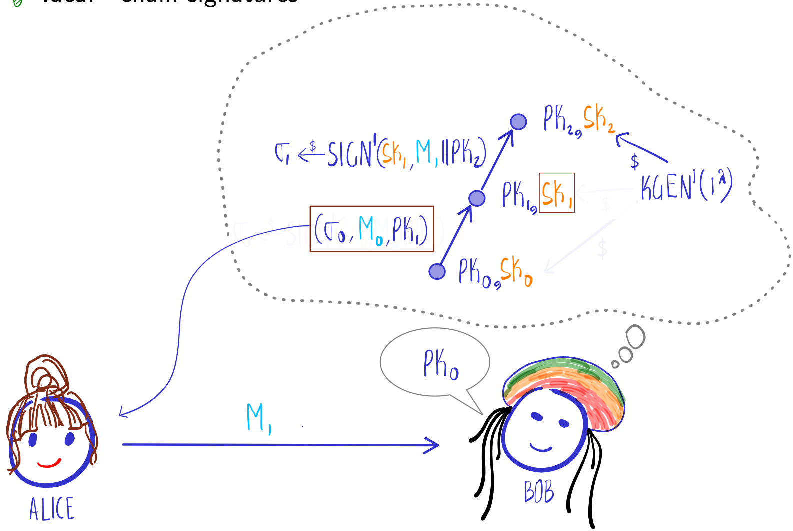

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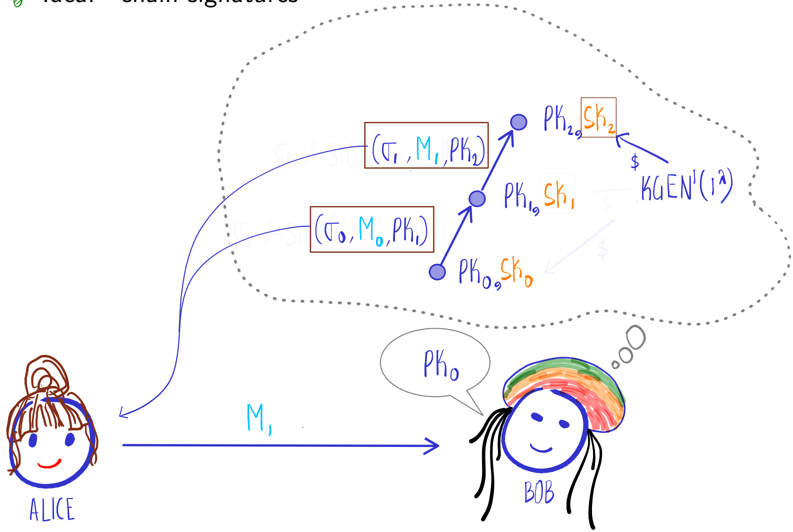

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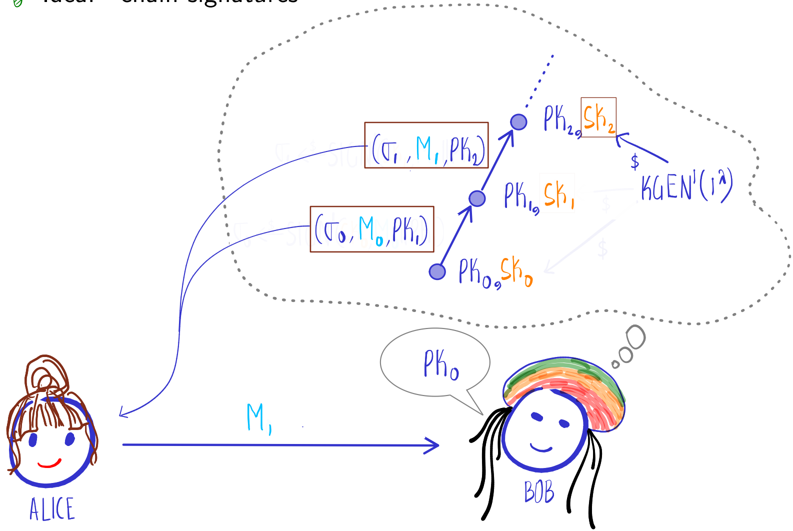

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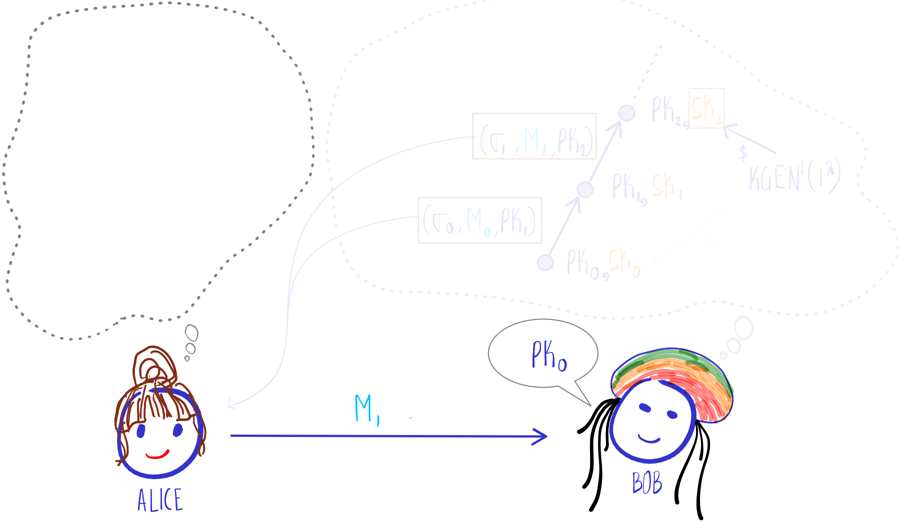

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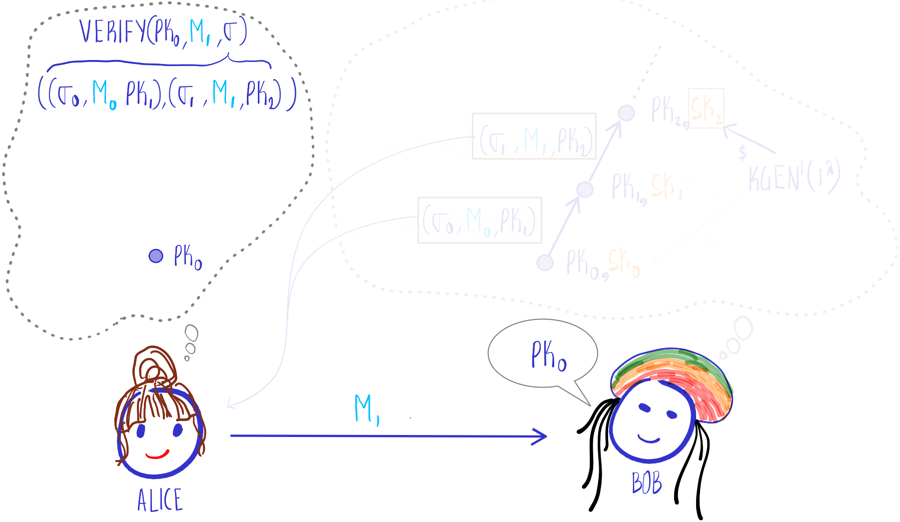

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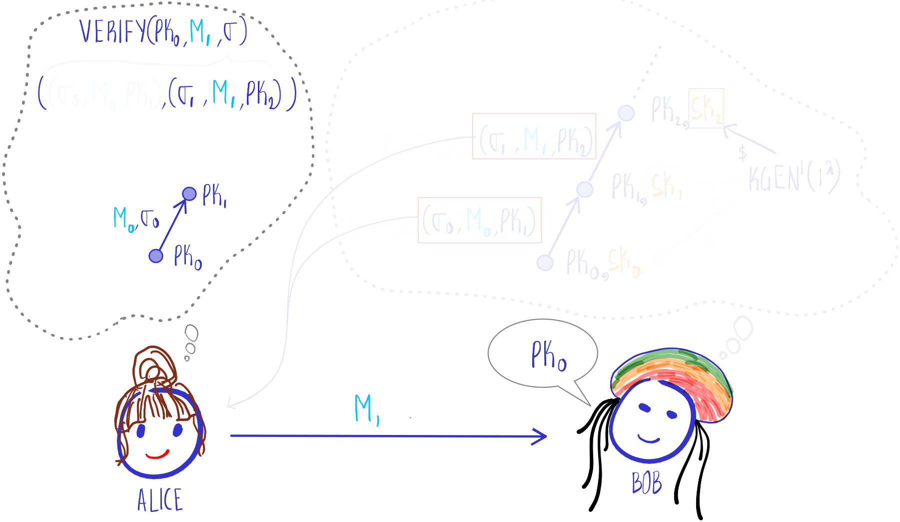

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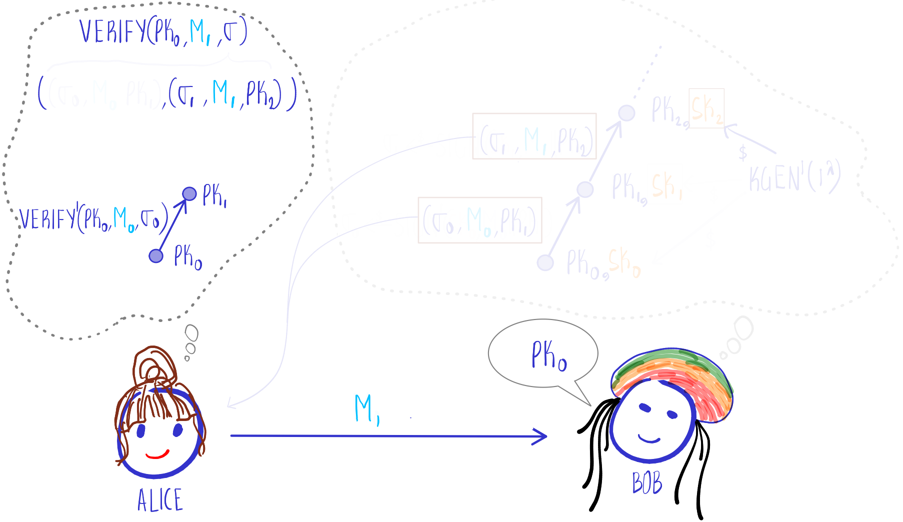

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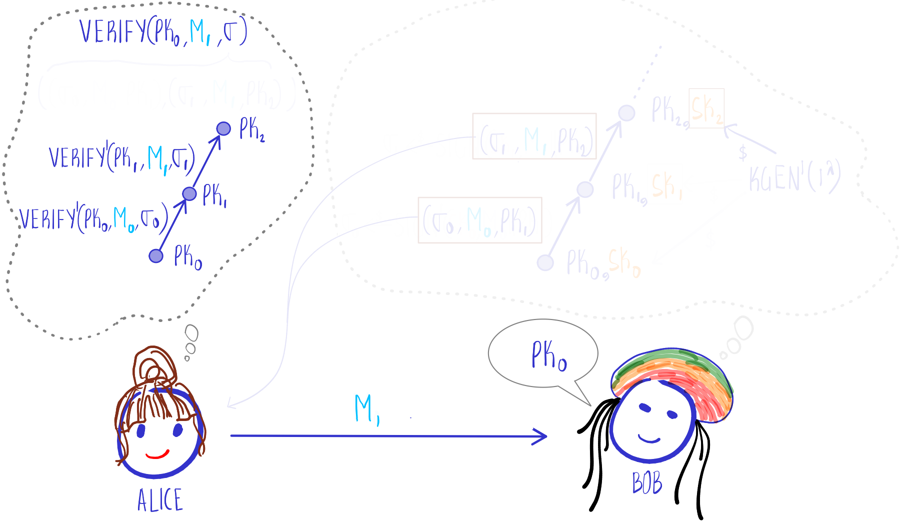

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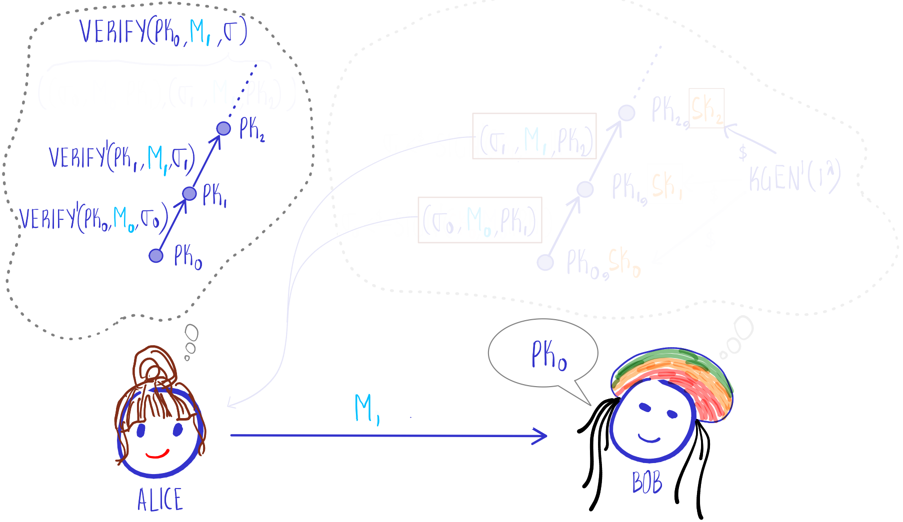

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ADVERSARY

REDUCTION

$\square$

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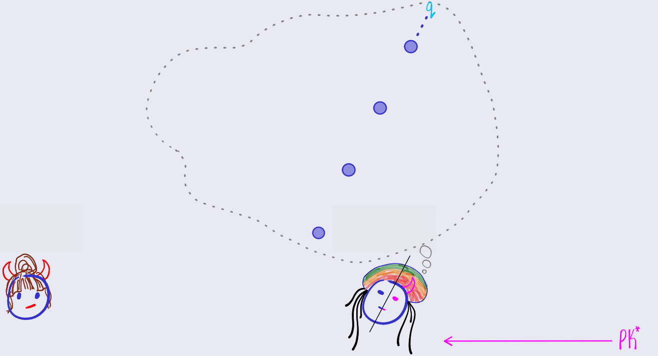

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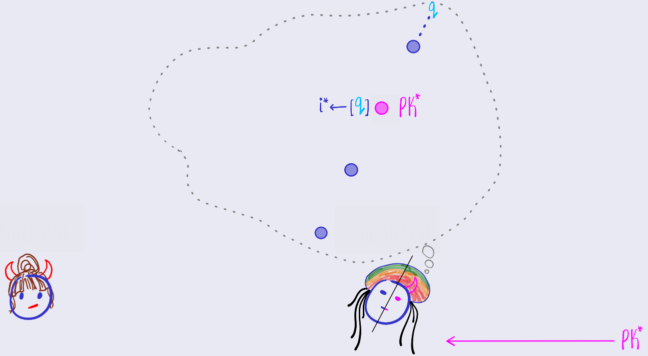

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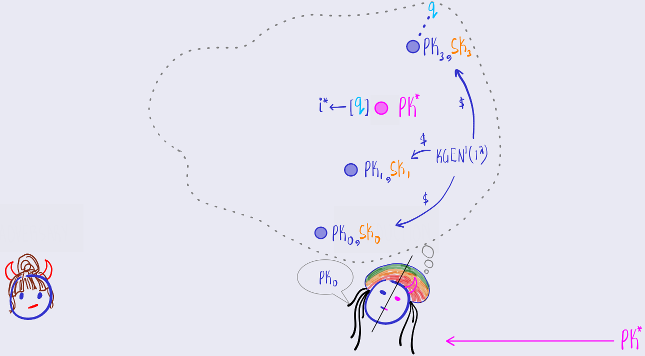

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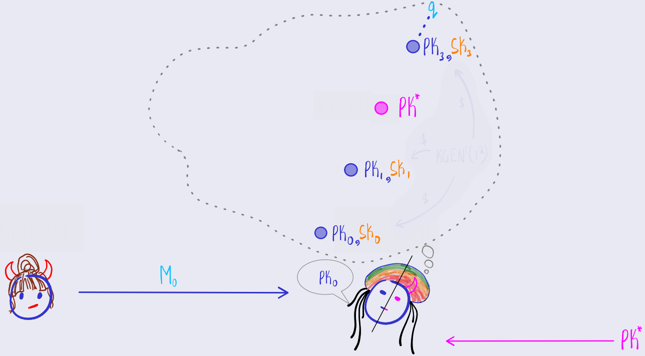

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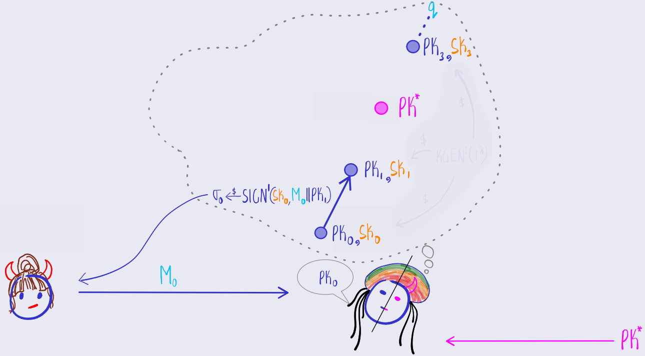

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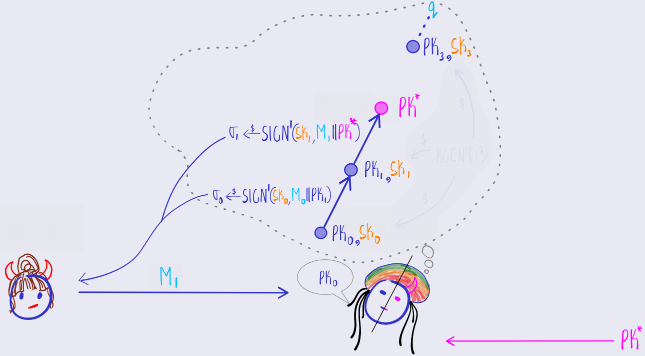

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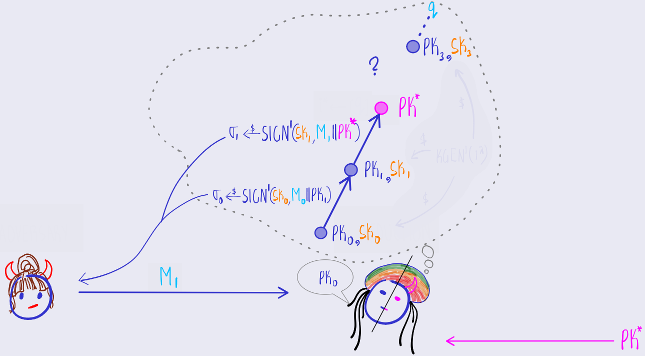

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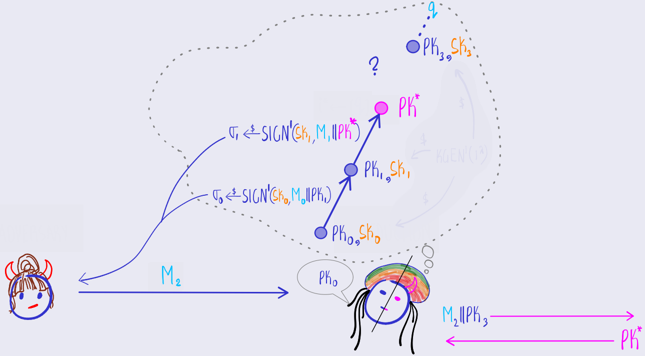

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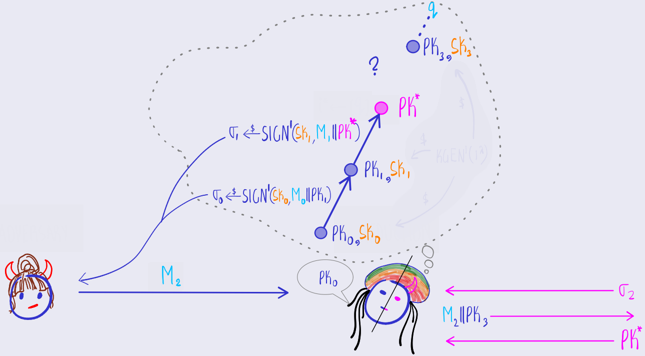

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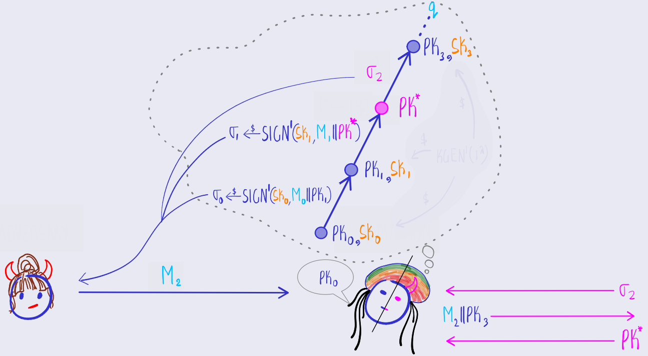

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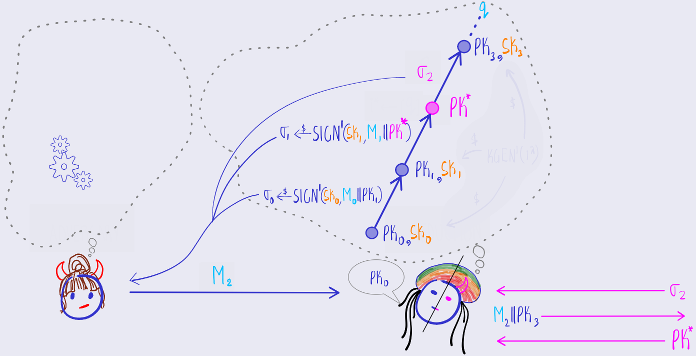

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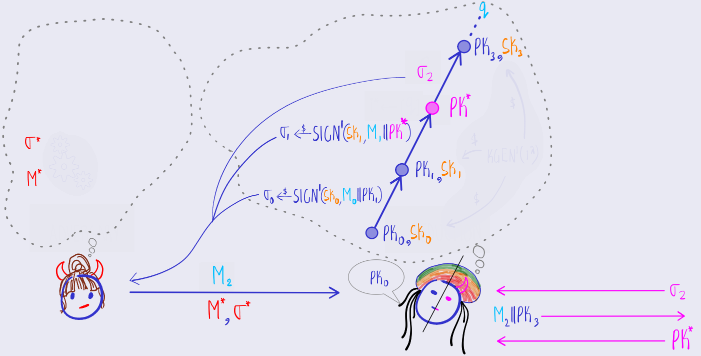

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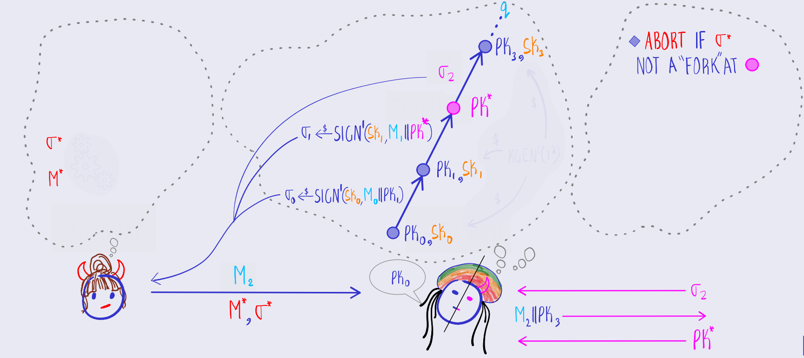

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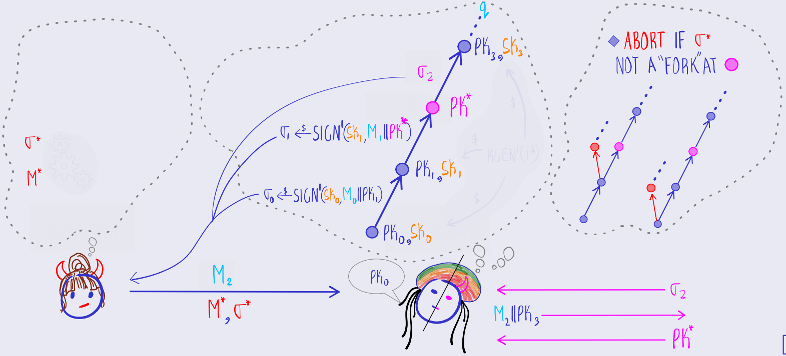

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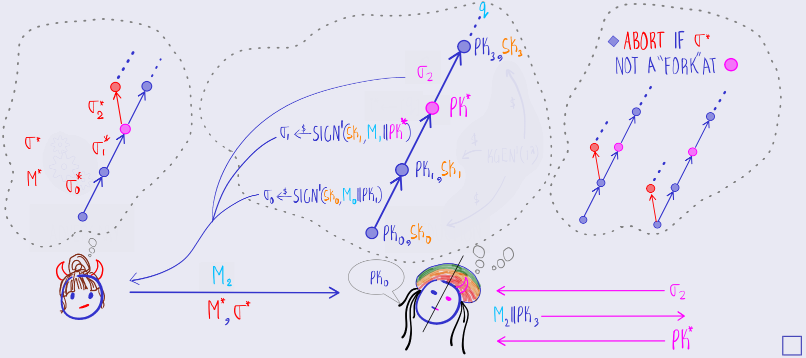

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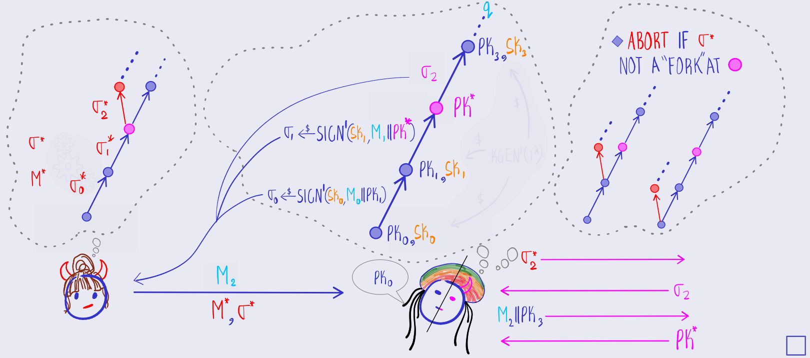

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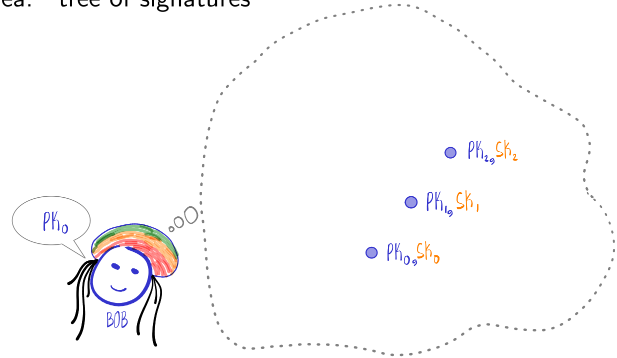

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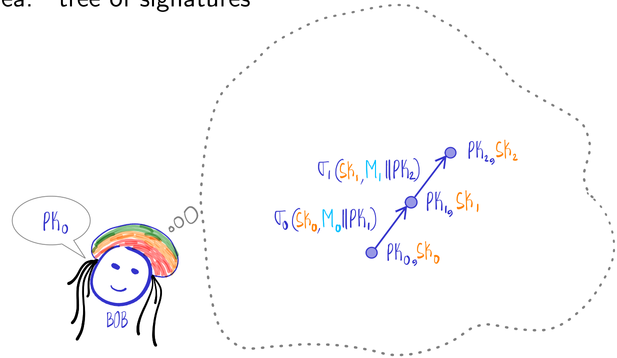

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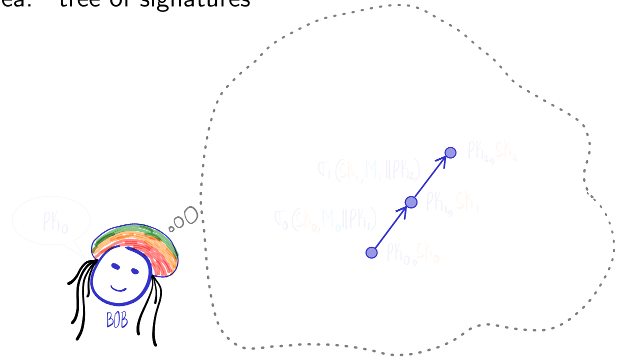

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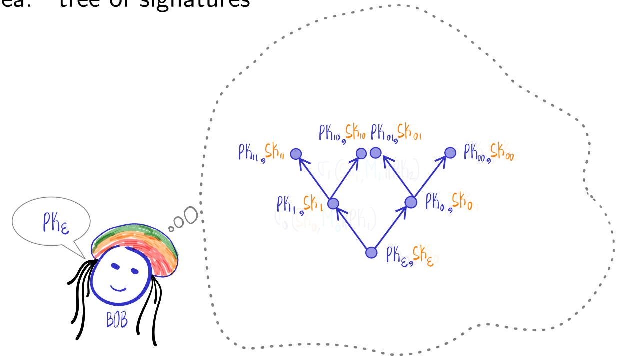

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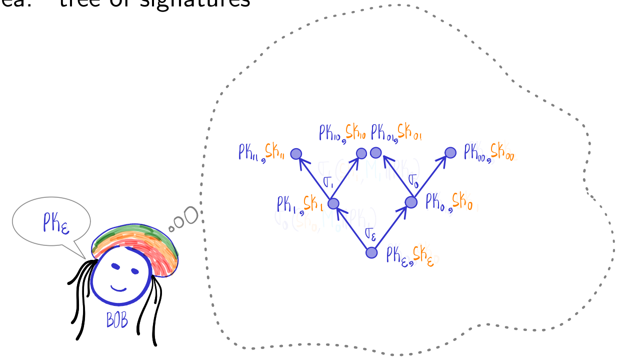

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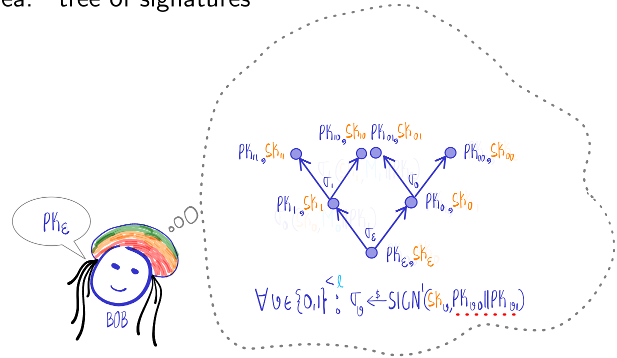

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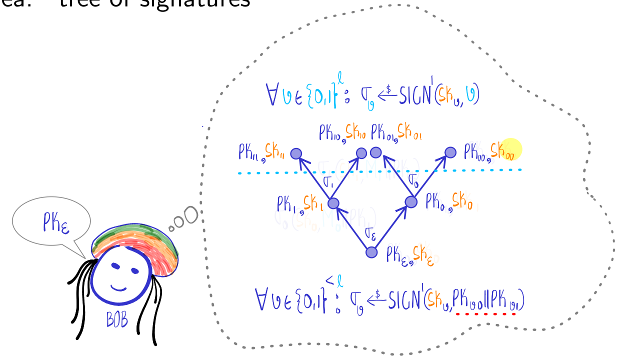

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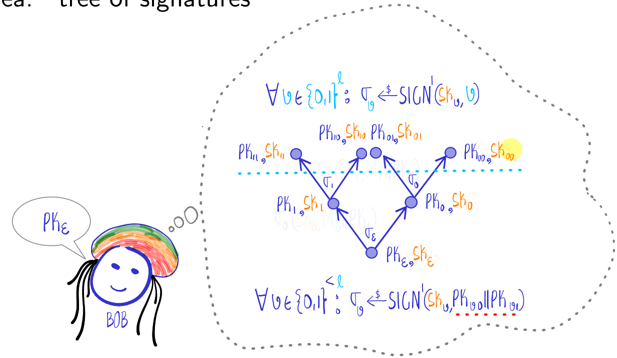

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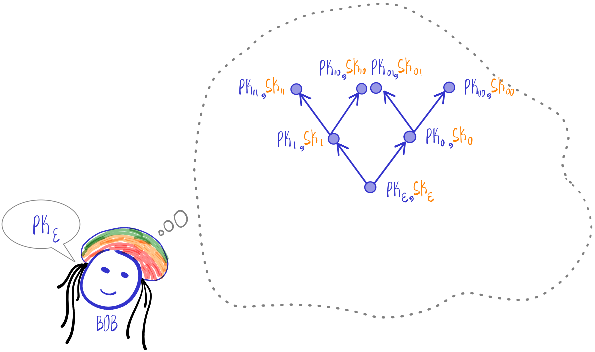

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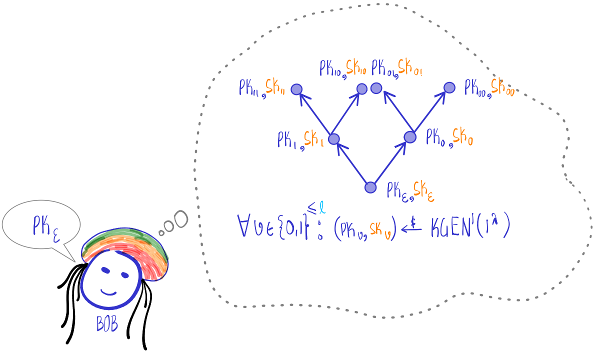

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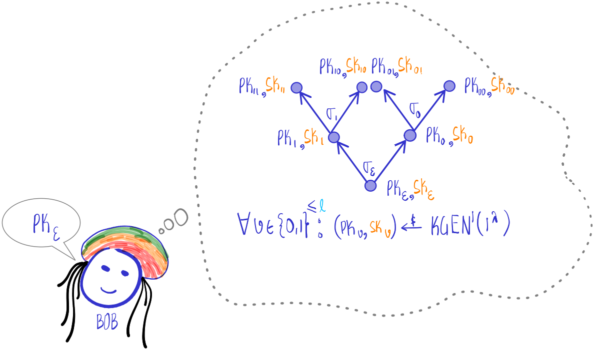

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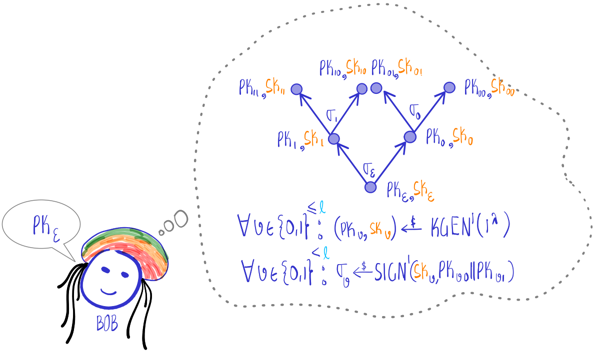

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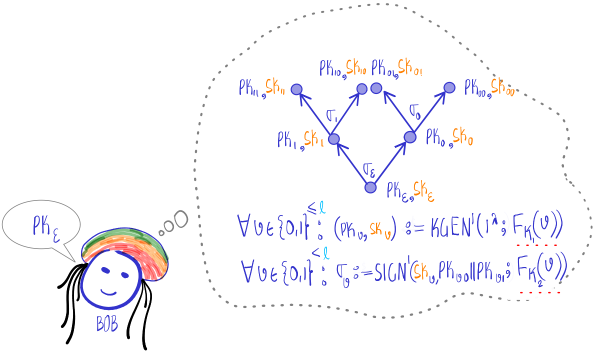

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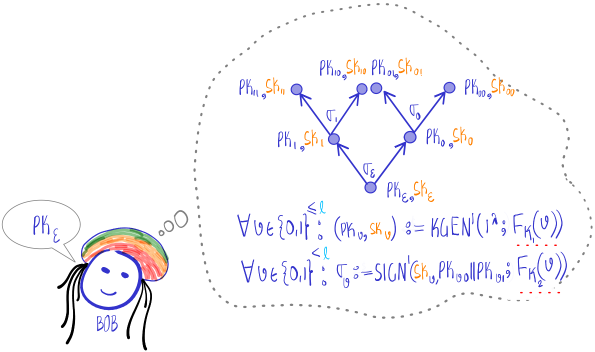

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Plan for this Session

Digital Signature: Syntax and Modelling Security

One-Time Signatures

Many-Time (Stateful) Signatures

Efficient Signatures via Hash-and-Sign

Wrapping Up

Efficient Signatures via Hash-then-Invert...

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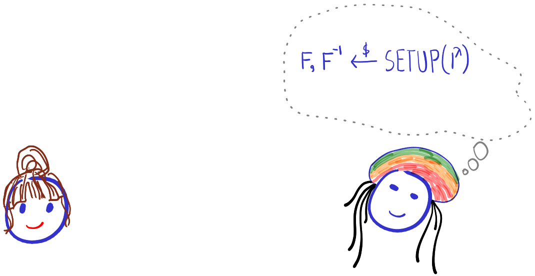

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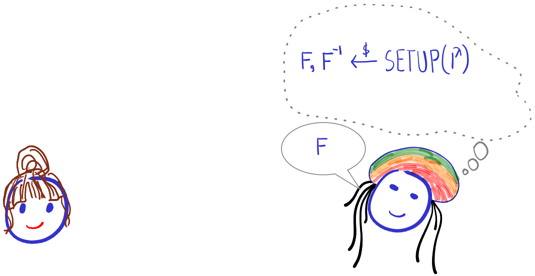

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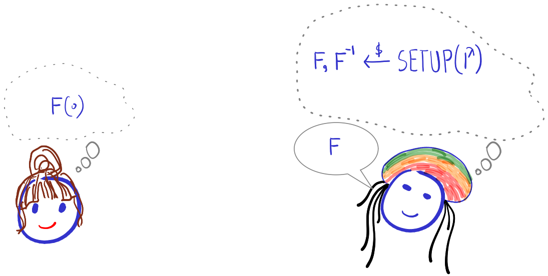

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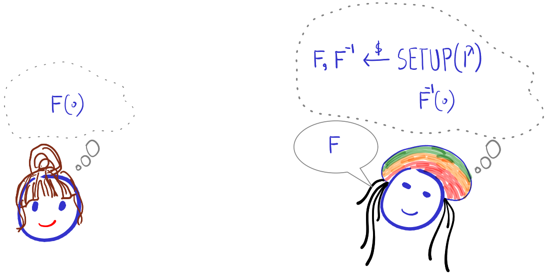

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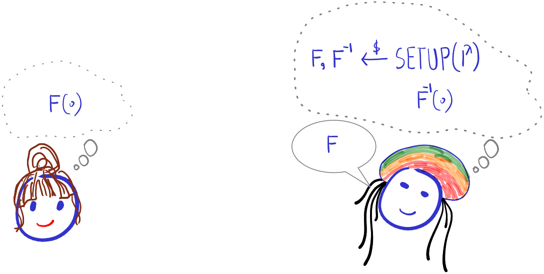

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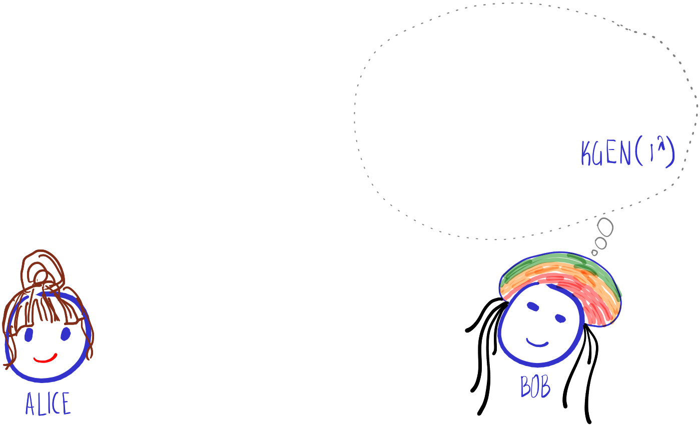

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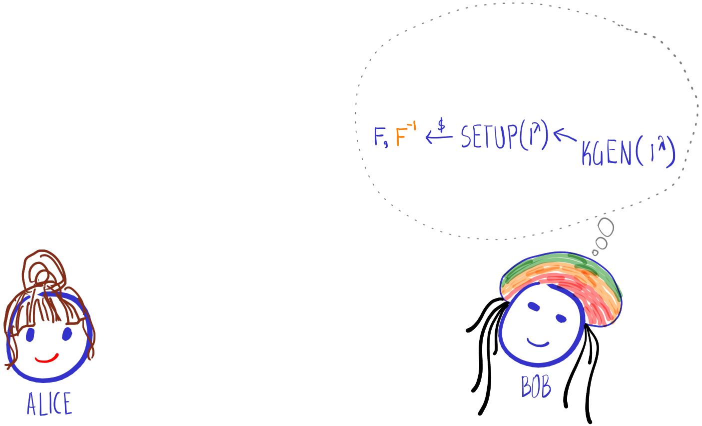

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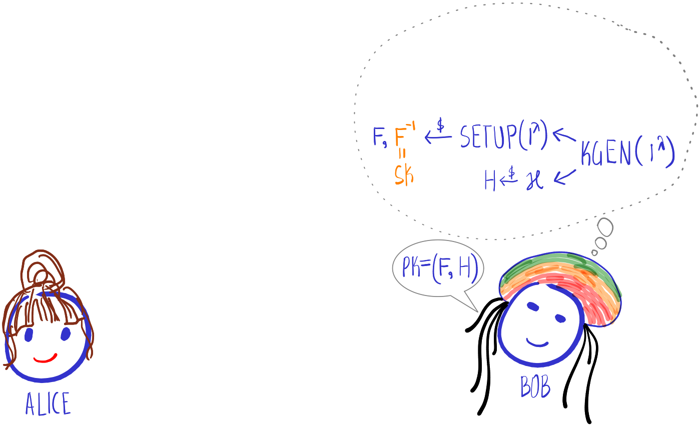

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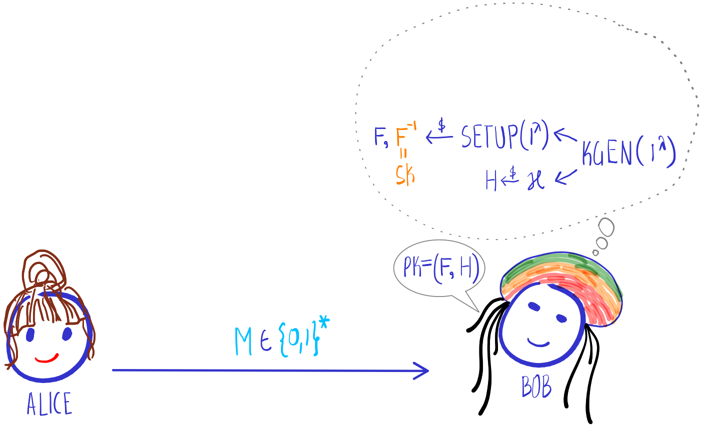

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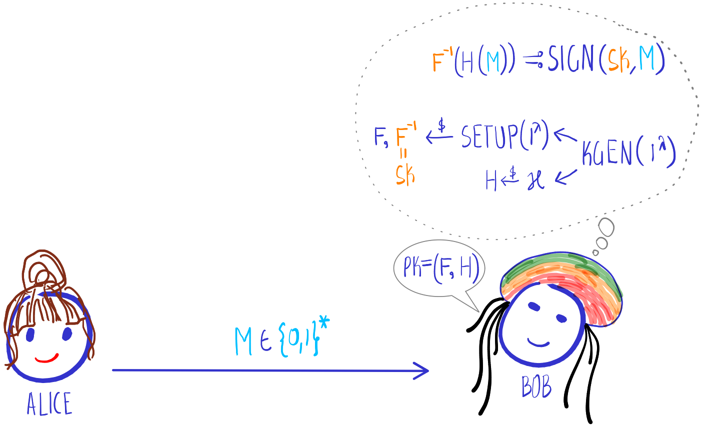

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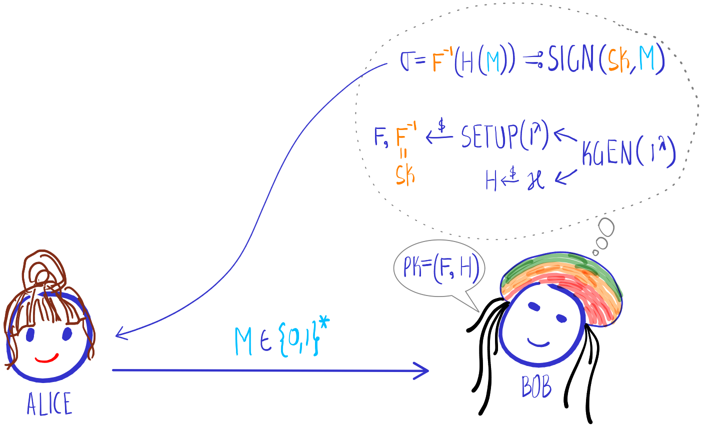

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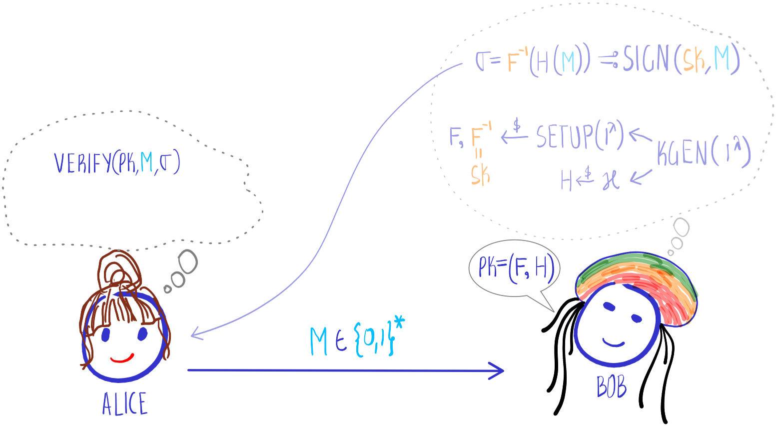

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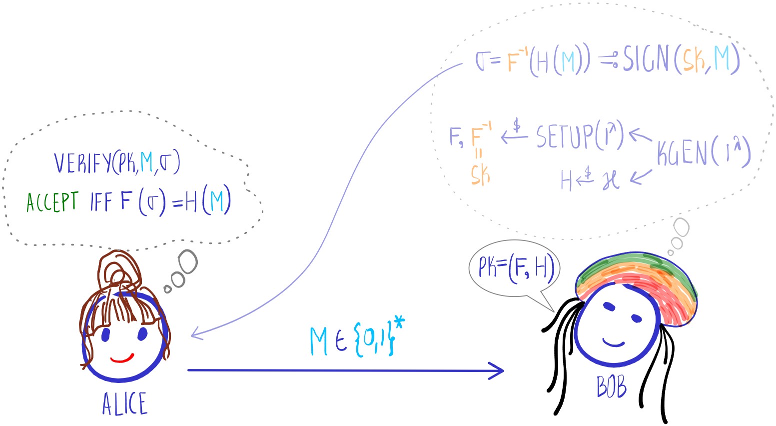

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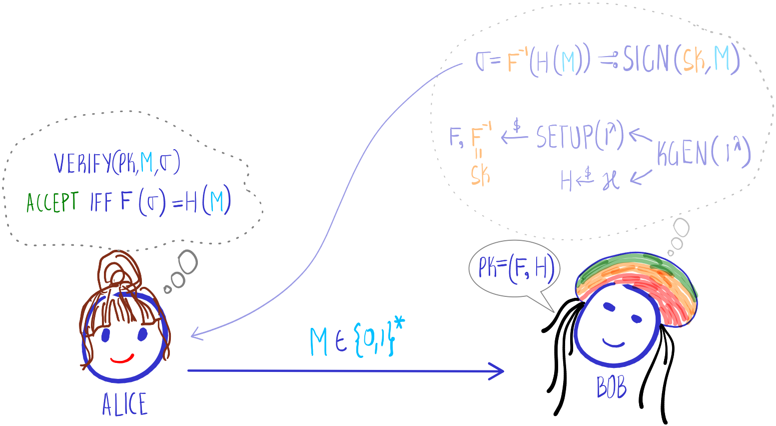

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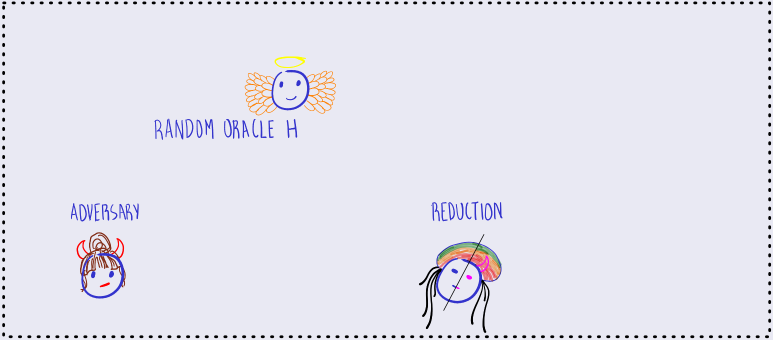
Proof sketch: random oracle programming.



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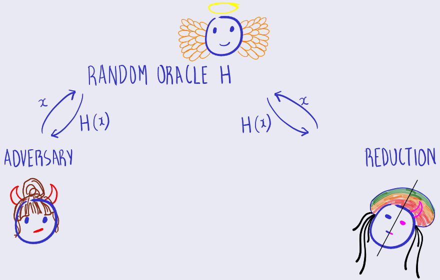

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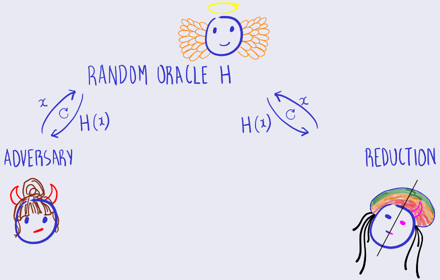

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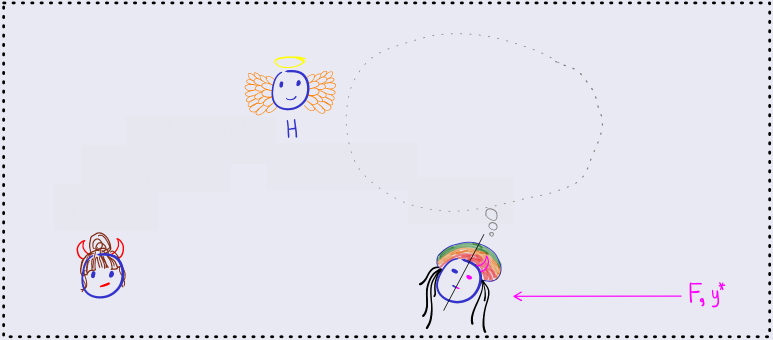

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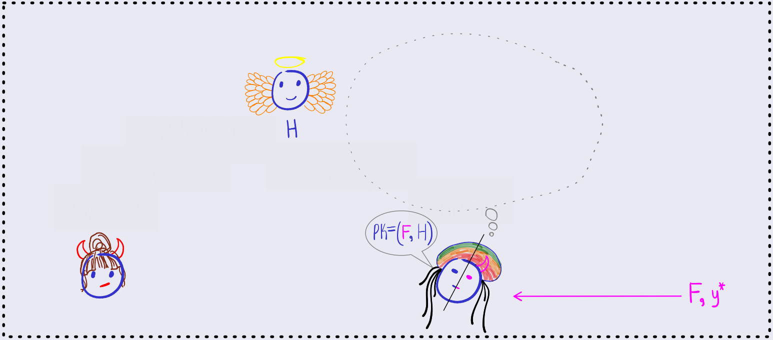

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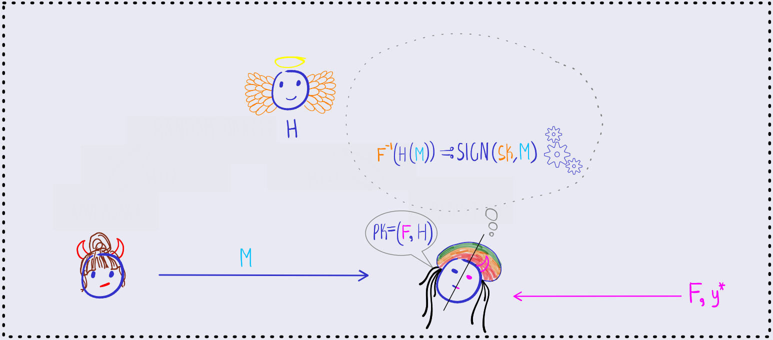

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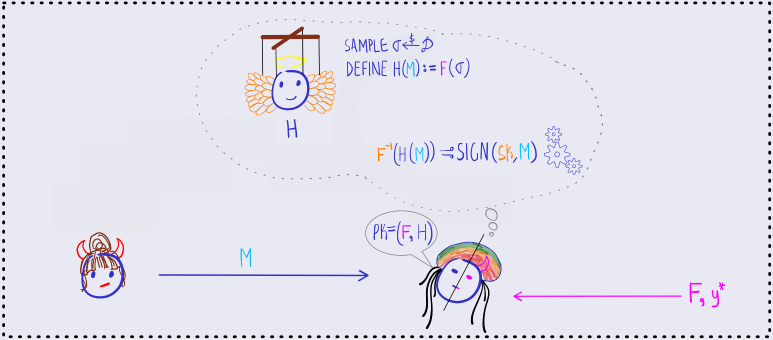

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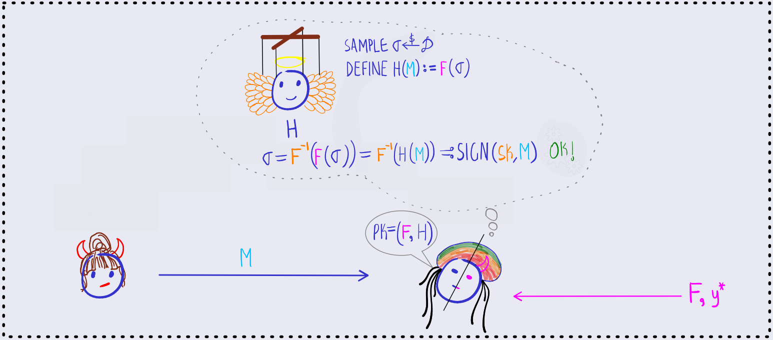

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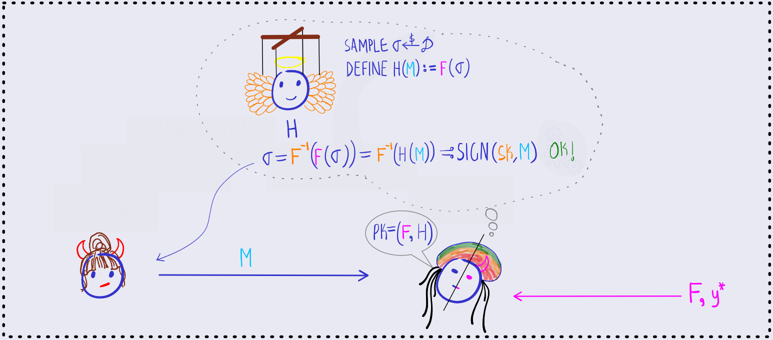

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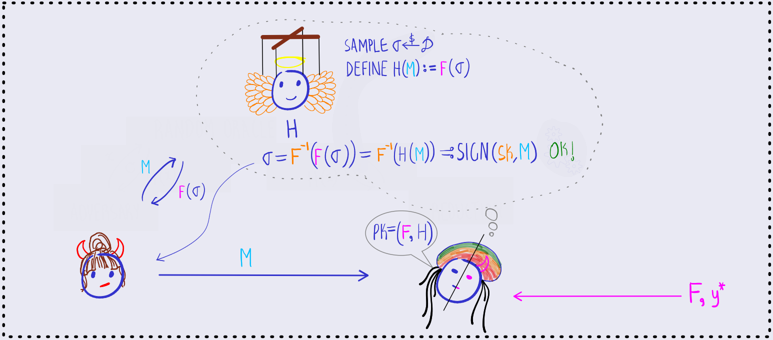

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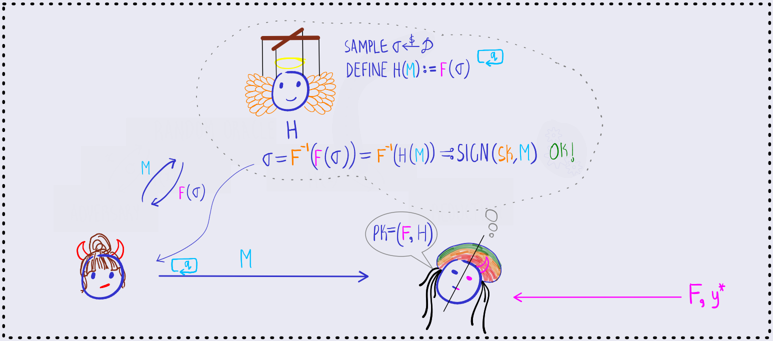

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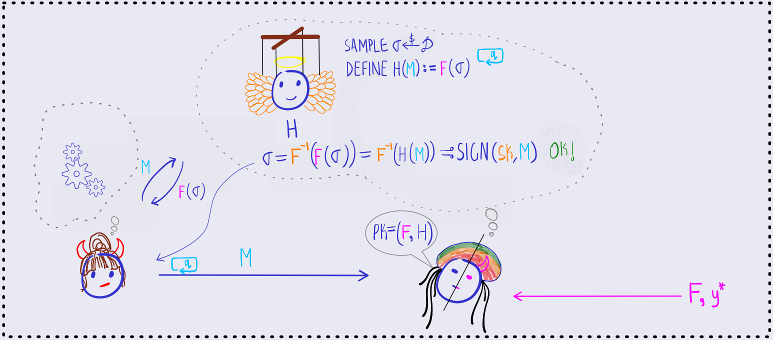

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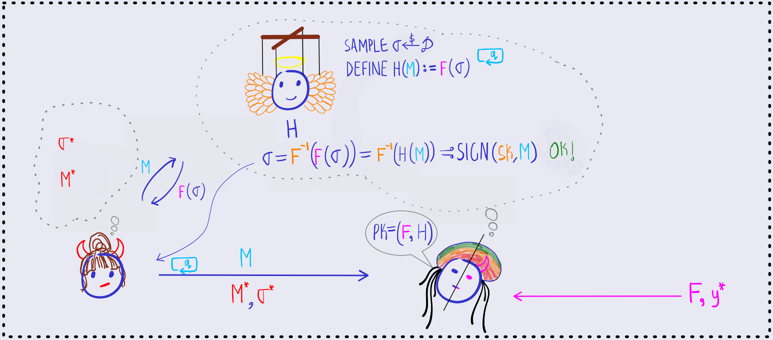

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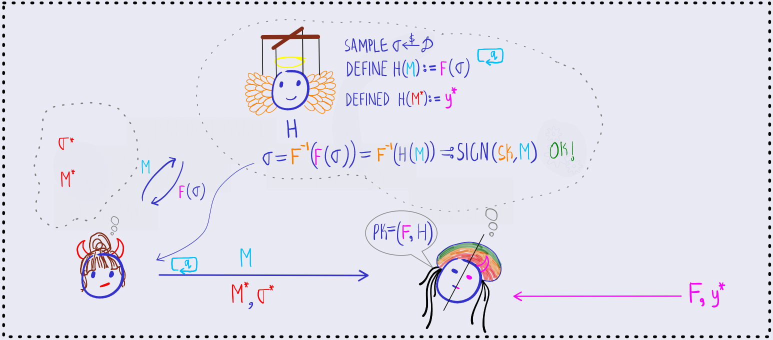

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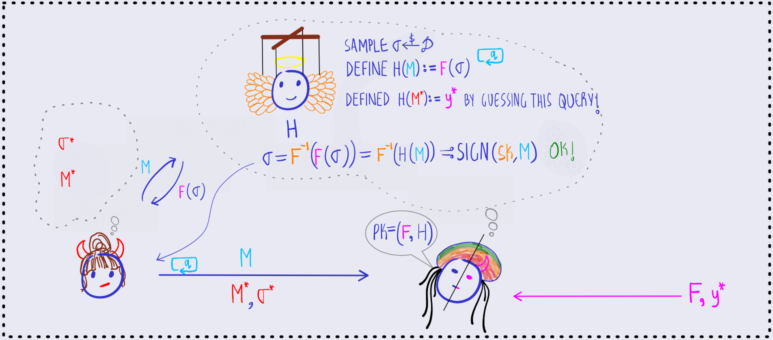

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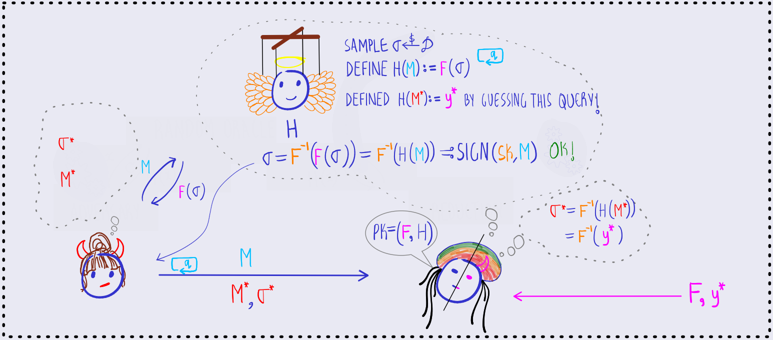

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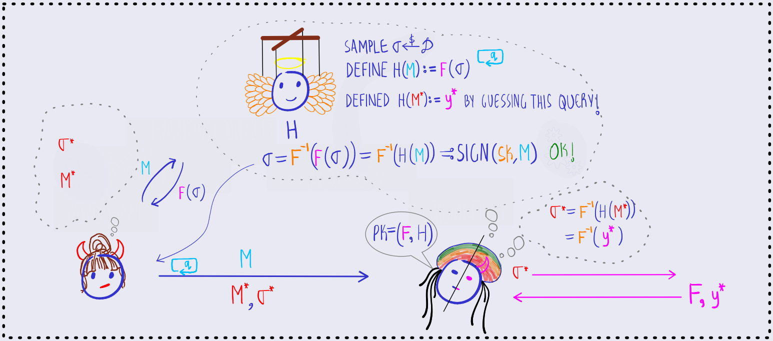

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Plan for this Session

Digital Signature: Syntax and Modelling Security

One-Time Signatures

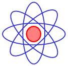
Many-Time (Stateful) Signatures

Efficient Signatures via Hash-and-Sign

Wrapping Up

To Recap

- Constructions:
  1. Theoretical construction of signature using OWF and CRHF
     - CRHF can be replaced with *universal one-way* hash function (UOWHF), which can be constructed from OWF
     - Can be shown to be quantum secure



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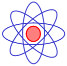

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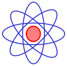

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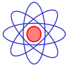

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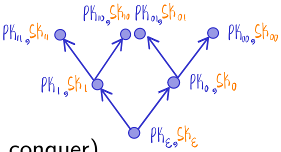

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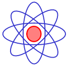

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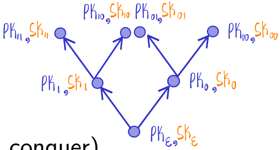

- Takeaways:
  - Constructive:
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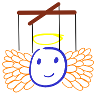
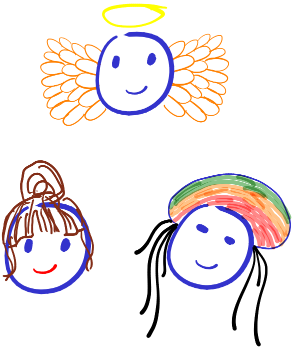
  - Proof techniques:
    - "Plug and pray"
    - Random oracle programming



Thank You for Your Attention! More Questions?



References

1. Digital signature and its security models were formally studied in [GMR88]
2. Lamport's OTS is from [Lam79]
3. The stateful many-time signature is from [KL21], and is a in spirit with Merkle's signatures [Mer90]
4. The "hash-then-invert" paradigm in random-oracle model was studied in [BR93]

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